and

$$\omega_j r_j \sin(\theta_i - \theta_j) = -\dot{r}_j \cos(\theta_i - \theta_j) - \dot{r}_i + \dot{b}_x \cos\theta_i + \dot{b}_y \sin\theta_i \tag{2.85}$$

Fifth Case

The loop-closure Equation 2.45 is differentiated under the assumption that all parameters, except γ and β, are time dependent.

$$\begin{aligned} &\dot{r}_i [\cos\theta_i, \sin\theta_i]^T + r_i[-\sin\theta_i, cos\theta_i]^T \omega_i \\ &+ \dot{r}_j[\cos(\theta_i - \gamma), \sin(\theta_i - \gamma)]^T + r_j[-\sin(\theta_i - \gamma), \cos(\theta_i - \gamma)]^T \omega_\iota \\ &+ \dot{r}_k[\cos(\theta_i - \beta), \sin(\theta_i - \beta)]^T + r_k[-\sin(\theta_i - \beta), \cos(\theta_i - \beta)]^T \omega_\iota = [\dot{b}_x, \dot{b}_y]^T \end{aligned} \tag{2.86}$$

Recall that the unknowns in this case are $\omega_i(t)$ and $\dot{r}_j(t)$. Collect similar terms in the latter equation.

$$\begin{aligned} &\dot{r}_i[\cos\theta_i, \sin\theta_i]^T + [d_x, d_y]^T \omega_i + \dot{r}_j[\cos(\theta_i - \gamma), \sin(\theta_i - \gamma)]^T \\ &+ \dot{r}_k[\cos(\theta_i - \beta), \sin(\theta_i - \beta)]^T = [\dot{b}_x, \dot{b}_y]^T \end{aligned} \tag{2.87}$$

where it is denoted

$$d_x = -r_i \sin\theta_i - r_j \sin(\theta_i - \gamma) - r_k \sin(\theta_i - \beta) \tag{2.88}$$

and

$$d_y = r_i \cos\theta_i + r_j \cos(\theta_i - \gamma) + r_k \cos(\theta_i - \beta) \tag{2.89}$$

Now the two unknowns can be found in the usual way by identifying the unit vectors perpendicular to the vectors $[\cos(\theta_i - \gamma), \sin(\theta_i - \gamma)]^T$ and $[d_x, d_y]^T$, and premultiplying Equation 2.87 by these vectors (note that the vector perpendicular to the latter vector is $[-d_{4444}, d_x]^T$). As a result, the following expressions are obtained:

$$\begin{aligned} (-d_x \sin(\theta_i - \gamma) + d_y \cos(\theta_i - \gamma))\omega_i &= -\dot{r}_i \sin\gamma - \dot{r}_k \sin(\gamma - \beta) \\ &\quad -\dot{b}_x \sin(\theta_i - \gamma) + \dot{b}_y \cos(\theta_i - \gamma) \end{aligned} \tag{2.90}$$

and

$$\begin{aligned} (-d_y \cos(\theta_i - \gamma) + d_x \sin(\theta_i - \gamma))\dot{r}_j &= -\dot{r}_i + (-d_y \cos\theta_i + d_x \sin\theta_i) \\ &\quad -\dot{r}_k(-d_y \cos(\theta_i - \beta) + d_x \sin(\theta_i - \beta)) + (-d_y \dot{b}_x + d_x \dot{b}_y) \end{aligned} \tag{2.91}$$