Universidad Tecnológica de Panamá
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Mecanismos
Parcial # 1: "Análisis cinemático de eslabonamientos que presentan juntas primarias"

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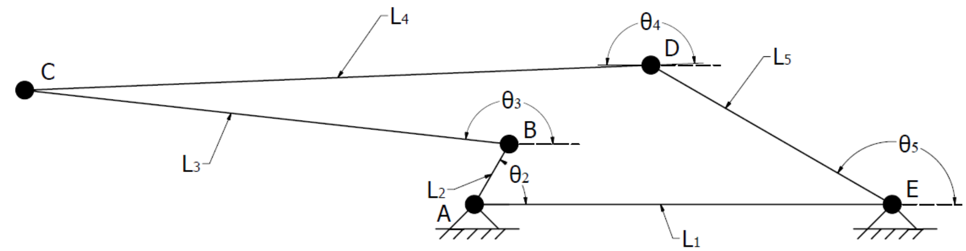
Grupo: 1N131

I. Resuelva el siguiente problema (100 puntos). Lea atentamente, siga las siguientes instrucciones, y enuncie sus suposiciones*.

Problema # 1.

Considere el mecanismo de cinco barras mostrado en la figura # 1. Haga lo siguiente:

a. Identifique los eslabones y las juntas existentes.



Eslabones: L_1, L_2, L_3, L_4, L_5.

Juntas de rotula: A, B, C, D, E.

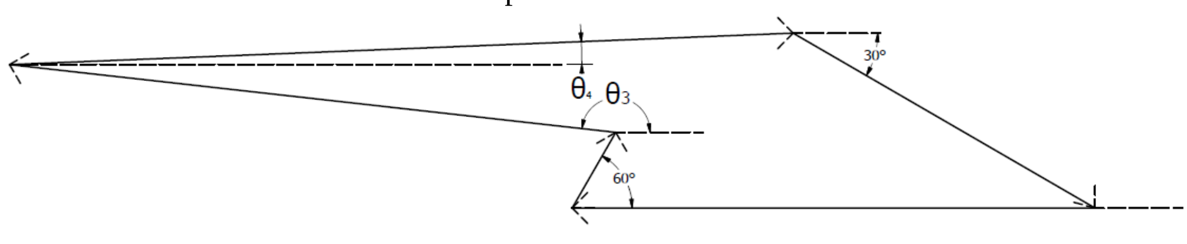
b. Determine el número de grados de libertad.

$$M = 3(n-1) - 2j_p - j_h$$

$$n = 5, j_p = 5, j_h = 0$$

$$M = 2$$

c. Deduzca la ecuación de laso cerrado para este mecanismo.



$$\sum_{i=1}^{5} \boldsymbol{r_i} = 0$$

$$|r_1|(\cos\theta_1, \sin\theta_1)^T + |r_2|(\cos\theta_2, \sin\theta_2)^T + |r_3|(\cos\theta_3, \sin\theta_3)^T + |r_4|(\cos\theta_4, \sin\theta_4)^T + |r_5|(\cos\theta_5, \sin\theta_5)^T = 0$$

d. Si se conoce el valor de $L_1, L_2, L_3, L_4, L_5, \theta_2, \theta_5$; determine el conjunto de ecuaciones no lineales necesarias para encontrar expresiones explicitas para θ_3 y para θ_4.

$$|r_3|(\cos\theta_3, \sin\theta_3)^T + |r_4|(\cos\theta_4, \sin\theta_4)^T = -[|r_1|(\cos\theta_1, \sin\theta_1)^T + |r_2|(\cos\theta_2, \sin\theta_2)^T + |r_5|(\cos\theta_5, \sin\theta_5)^T]$$

Sea:

$$\boldsymbol{b} = |b|(\cos\alpha, \sin\alpha)^T = \left(b_x, b_y\right)^T = -[|r_1|(\cos\theta_1, \sin\theta_1)^T + |r_2|(\cos\theta_2, \sin\theta_2)^T + |r_5|(\cos\theta_5, \sin\theta_5)^T]$$

$$|r_3|(\cos\theta_3, \sin\theta_3)^T + |r_4|(\cos\theta_4, \sin\theta_4)^T = |b|(\cos\alpha, \sin\alpha)^T$$

Si la expresión anterior se multiplica primero por un vector unitario perpendicular al vector $\boldsymbol{b}$, $\boldsymbol{u}_{\perp \boldsymbol{b}} = (-\sin\alpha, \cos\alpha)^T$, y luego por un vector unitario paralelo al vector $\boldsymbol{b}$, $\boldsymbol{u}_{\parallel \boldsymbol{b}} = (\cos\alpha, \sin\alpha)^T$; se tendrá:

$$|r_3|\sin(\alpha - \theta_3) + |r_4|\sin(\alpha - \theta_4) = 0$$

$$|r_3|\cos(\alpha - \theta_3) + |r_4|\cos(\alpha - \theta_4) = |b|$$

Y a partir de las expresiones anteriores se puede determinar θ_3, θ_4. Para las expresiones explicitas ver clase # 3.

e. A partir de las expresiones deducidas en el inciso anterior, para una configuración en donde $L_1 = 6\text{ in}, L_2 = 1\text{ in}, L_3 = 7\text{ in}, L_4 = 9\text{ in}, L_5 = 4\text{ in}, \theta_2 = 60°$, $\theta_5 = 30° + 2\theta_2$; determine el valor que tendrá θ_3 [°] y θ_4 [°].

$$b_x = -\left[[6(\cos(180°)) + 1(\cos(60°)) + 4(\cos(330°))]\right]$$
$$b_x \cong 2.035898\ in$$
$$b_y = -\left[[6(\sin(180°)) + 1(\sin(60°)) + 4(\sin(330°))]\right]$$
$$b_y \cong 1.133975\ in$$
$$|b| \cong 2.330403\ in$$
$$\alpha \cong 29.1174°$$

$$7\sin(29.1174° - \theta_3) + 9\sin(29.1174° - \theta_4) = 0$$

$$7\cos(29.1174° - \theta_3) + 9\cos(29.1174° - \theta_4) = 2.330403$$

$$\theta_3 \cong 173.642°$$
$$\theta_4 \cong 2.285°$$

Respecto a la referencia original:

$$\theta_3 \cong 173.642°$$
$$\theta_4 \cong 180° + 2.285° \cong 182.285°$$

f. A partir del análisis de posición anterior y empleando el análisis de velocidad determine $\dot{\theta}_3$ [rad/s] y $\dot{\theta}_4$ [rad/s] si se conoce que $\dot{\theta}_2 = 10\text{ rad/s}$.

$$b_x = -(|r_1(t)|\cos(\theta_1(t)) + |r_2(t)|\cos(\theta_2(t)) + |r_1(t)|\cos(\theta_1(t)) + |r_5(t)|\cos(\theta_5(t)))$$

$$\dot{b}_x = -\left(|\dot{r}_1(t)|\cos(\theta_1(t)) - |r_1(t)|\dot{\theta}_1(t)\sin\left(\theta_1(t)\right) + |\dot{r}_2(t)|\cos\left(\theta_2(t)\right) - |r_2(t)|\dot{\theta}_2(t)\sin\left(\theta_2(t)\right) + |\dot{r}_5(t)|\cos\left(\theta_5(t)\right) - |r_5(t)|\dot{\theta}_5(t)\sin\left(\theta_5(t)\right)\right)$$

$$\dot{b}_x = -\left(-|r_2(t)|\dot{\theta}_2(t)\sin\left(\theta_2(t)\right) - |r_5(t)|\dot{\theta}_5(t)\sin\left(\theta_5(t)\right)\right)$$

$$\dot{b}_x = -(-1(10)\sin(60°) - 4(20)\sin(330°)) \cong -31.339746\ in/s$$

$$\dot{b}_y = -\left(|\dot{r}_1(t)|\sin(\theta_1(t)) + |r_1(t)|\dot{\theta}_1(t)\cos\left(\theta_1(t)\right) + |\dot{r}_2(t)|\sin\left(\theta_2(t)\right) + |r_2(t)|\dot{\theta}_2(t)\cos\left(\theta_2(t)\right) + |\dot{r}_5(t)|\sin(\theta_5(t)) + |r_5(t)|\dot{\theta}_5(t)\cos\left(\theta_5(t)\right)\right)$$

$$\dot{b}_y = -\left(|r_2(t)|\dot{\theta}_2(t)\cos\left(\theta_2(t)\right) + |r_5(t)|\dot{\theta}_5(t)\cos\left(\theta_5(t)\right)\right)$$

$$\dot{b}_y = -(1(10)\cos(60°) + 4(20)\cos(330°)) \cong -74.282032\ in/s$$

$$\dot{\theta}_3(t) = \frac{\dot{b}_x\cos\theta_4(t) + \dot{b}_y\sin\theta_4(t) - |\dot{r}_4(t)| - |\dot{r}_3(t)|[\cos(\theta_4 - \theta_3)]}{|r_3(t)|[\sin(\theta_4 - \theta_3)]}$$

$$\dot{\theta}_3(t) \cong \frac{-34.276466}{(7)[\sin(2.285 - 173.642)]} \cong 32.585\frac{rad}{s}\ Antihorario$$

$$\dot{\theta}_4(t) = \frac{\dot{b}_x\cos\theta_3(t) + \dot{b}_y\sin\theta_3(t) - |\dot{r}_3(t)| - |\dot{r}_4(t)|[\cos(\theta_3 - \theta_4)]}{|r_4(t)|[\sin(\theta_3 - \theta_4)]}$$

$$\dot{\theta}_4(t) = \frac{22.920962}{9[\sin(173.642 - 2.285)]} \cong 16.948\frac{rad}{s}\ Antihorario$$

g. A partir del análisis de posición y de velocidad anterior, y empleando el análisis de aceleración determine $\ddot{\theta}_3$ [rad/s^2] y $\ddot{\theta}_4$ [rad/s^2]. Considere que la velocidad angular del eslabón 2 de 10 rad/s es constante.

$$\ddot{b}_x = -\frac{d}{dt}\left(-|r_2(t)|\dot{\theta}_2(t)\sin\left(\theta_2(t)\right) - |r_5(t)|\dot{\theta}_5(t)\sin\left(\theta_5(t)\right)\right)$$

$$\ddot{b}_x = -\left[-\left(|\dot{r}_2(t)|\dot{\theta}_2(t) + |r_2(t)|\ddot{\theta}_2(t)\right)\sin\left(\theta_2(t)\right) - |r_2(t)|\dot{\theta}_2(t)^2\cos\left(\theta_2(t)\right)\right] - \left[-\left(|\dot{r}_5(t)|\dot{\theta}_5(t) + |r_5(t)|\ddot{\theta}_5(t)\right)\sin\left(\theta_5(t)\right) - |r_5(t)|\dot{\theta}_5(t)^2\cos\left(\theta_5(t)\right)\right]$$

$$\ddot{\theta}_2 = 0 \to \ddot{\theta}_5 = 2\ddot{\theta}_2 = 0$$

$$\ddot{b}_x = -\left[-|r_2(t)|\dot{\theta}_2(t)^2\cos\left(\theta_2(t)\right)\right] - \left[-|r_5(t)|\dot{\theta}_5(t)^2\cos\left(\theta_5(t)\right)\right]$$

$$\ddot{b}_x = [(1)(10)^2 \cos(60°)] + [(4)(20)^2 \cos(330°)] \cong 1435.640646\ in/s^2$$

$$\dot{b}_y = -\frac{d}{dt}\left(|r_2(t)|\dot{\theta}_2(t)\cos\big(\theta_2(t)\big) + |r_5(t)|\dot{\theta}_5(t)\cos\big(\theta_5(t)\big)\right)$$

$$\ddot{b}_y = -\left[-|r_2(t)|\dot{\theta}_2(t)^2 \sin\big(\theta_2(t)\big)\right] - \left[-|r_5(t)|\dot{\theta}_5(t)^2 \sin\big(\theta_5(t)\big)\right]$$

$$\ddot{b}_y = [(1)(10)^2 \sin(60°)] + [(4)(20)^2 \sin(330°)] \cong -713.397460\ in/s^2$$

$$\ddot{\theta}_3(t) = \frac{d}{dt}\left(\frac{\dot{b}_x \cos\theta_4(t)}{|r_3(t)|[\sin(\theta_4 - \theta_3)]} + \frac{\dot{b}_y \sin\theta_4(t)}{|r_3(t)|[\sin(\theta_4 - \theta_3)]}\right) = \frac{d}{dt}(f_1 + f_2)$$

$$\frac{d}{dt}(f_1) =$$

$$\frac{\big(\ddot{b}_x \cos\theta_4(t) - \dot{b}_x\dot{\theta}_4(t)\sin\theta_4(t)\big)|r_3(t)|[\sin(\theta_4 - \theta_3)] - \big(\dot{b}_x \cos\theta_4(t)\big)(|r_3(t)|)\big(\dot{\theta}_4 - \dot{\theta}_3\big)\cos(\theta_4 - \theta_3)}{(|r_3(t)|[\sin(\theta_4 - \theta_3)])^2}$$

$$\frac{d}{dt}(f_1) =$$

$$\frac{(1455.675821)(7)[-0.150277] - (-31.314827)(7)(-15.637)(-0.988644)}{(1.106581)}$$

$$\frac{d}{dt}(f_1) \cong 1678.578105\ rad/s^2$$

$$\frac{d}{dt}(f_2) =$$

$$\frac{\big(\ddot{b}_y \sin\theta_4(t) + \dot{b}_y\dot{\theta}_4(t)\cos\theta_4(t)\big)|r_3(t)|[\sin(\theta_4 - \theta_3)] - \big(\dot{b}_y \sin\theta_4(t)\big)(|r_3(t)|)\big(\dot{\theta}_4 - \dot{\theta}_3\big)\cos(\theta_4 - \theta_3)}{(|r_3(t)|[\sin(\theta_4 - \theta_3)])^2}$$

$$\frac{d}{dt}(f_2) =$$

$$\frac{(-1286.374162)(7)[-0.150277] - (-2.961640)(7)(-15.637)(-0.988644)}{(1.106581)}$$

$$\frac{d}{dt}(f_2) \cong 1512.482082\ rad/s^2$$

$$\ddot{\theta}_3 \cong 3191.060\frac{rad}{s^2}$$

$$\ddot{\theta}_4(t) = \frac{d}{dt}\left(\frac{\dot{b}_x\cos\theta_3(t) + \dot{b}_y\sin\theta_3(t)}{|r_4(t)|[\sin(\theta_3-\theta_4)]}\right) = \frac{d}{dt}(f_3+f_4)$$

$$\frac{d}{dt}(f_3) = \frac{d}{dt}\left(\frac{\dot{b}_x\cos\theta_3(t)}{|r_4(t)|[\sin(\theta_3-\theta_4)]}\right)$$

$$\frac{d}{dt}(f_3) =$$

$$\frac{\left(\ddot{b}_x\cos\theta_3(t) - \dot{b}_x\dot{\theta}_3(t)\sin\theta_3(t)\right)|r_4(t)|[\sin(\theta_3-\theta_4)] - \left(\dot{b}_x\cos\theta_3(t)\right)(|r_4(t)|)\left(\dot{\theta}_3-\dot{\theta}_4\right)\cos(\theta_3-\theta_4)}{(|r_4(t)|[\sin(\theta_3-\theta_4)])^2}$$

$$\frac{d}{dt}(f_3) =$$

$$\frac{(-1313.721789)(9)[0.150277] - (31.146987)(9)(15.637)(-0.988644)}{(1.829246)}$$

$$\frac{d}{dt}(f_3) \cong 1397.751536\, rad/s^2$$

$$\frac{d}{dt}(f_4) = \frac{d}{dt}\left(\frac{\dot{b}_y\sin\theta_3(t)}{|r_4(t)|[\sin(\theta_3-\theta_4)]}\right)$$

$$\frac{d}{dt}(f_4) =$$

$$\frac{\left(\ddot{b}_y\sin\theta_3(t) + \dot{b}_y\dot{\theta}_3(t)\cos\theta_3(t)\right)|r_4(t)|[\sin(\theta_3-\theta_4)] - \left(\dot{b}_y\sin\theta_3(t)\right)(|r_4(t)|)\left(\dot{\theta}_3-\dot{\theta}_4\right)\cos(\theta_3-\theta_4)}{(|r_4(t)|[\sin(\theta_3-\theta_4)])^2}$$

$$\frac{d}{dt}(f_4) =$$

$$\frac{(2326.590576)(9)[0.150277] - (-8.226024)(9)(15.637)(-0.988644)}{(1.829246)}$$

$$\frac{d}{dt}(f_4) \cong 1094.533468\, rad/s^2$$

$$\ddot{\theta}_4 \cong 2492.29\frac{rad}{s^2}$$

Ambas aceleraciones anti horario.

Para el mecanismo de cinco barras anterior considere que los eslabones son rígidos y que el eslabón 1 está en reposo.