Linear Programming

Introduction

- It is a mathematical technique that allows the selection of the best course of action defining a program of feasible actions.
- The objective of LP is to assign resources that are scarce to different activities competing for them.
- The model that describes the different relationships among variables is composed of linear functions.
- George Dantzig is considered the father of LP

General formulation

- The main statement of the problem can be as follows:

  To optimize a dependent variable, expressed as linear function of n independent variables, subject to a series of constraints that are also linear function of the n independent variables.

General formulation

- The dependent variable is known as the **Objective Function**.
- This function is related to economic concepts such as earnings, income, time, cost, distance, etc.
- The independent variables are known as **decision variables.**

Standard formulation

$$\text{Optimize}: \quad f(x) = Z = \sum_{j=1}^{n} c_j x_j$$

Objective function

$$\text{subject to}: \quad g_i(x) = \sum_{j=1}^{n} a_{i,j} x_j \le b_i \;\; \forall\, i = 1, 2, ..., m$$

Constraints

Given that:

- f(.): objective function
- x_j : decision variables
- c_j : coefficient of the j^{th} decision variable in the objective function, for j = 1,…, n
- $a_{1,j}$: coefficient of the j^{th} decision variable in the i^{th} restriction, for i = 1,…, m
- b_i : constant or boundary of the i^{th} constraint.

The constraints:

- Linear programming is a response to situations that require the maximization or minimization of certain functions which are subject to limitations. These limitations are called **constraints**.
- There are three types of constraints:
  - $g(x) \leq b$
  - $g(x) \geq b$, or
  - $g(x) = b$
- Constraints type ≤ ensure that the use of resources do not exceed certain amount of it.
- Constraints type ≥ ensure that the use of certain resources will satisfy a minimum amount of it.
- Constraints type = ensure that the use of certain resource will be exactly as defined.

Model formulation: steps

- Good understanding of the problem
- Identify decision variables
- Define the objective function
- Define constraints
- Identify lower and upper boundaries of the decision variables

One example from Hillier: The Wyndor Case

- A window factory produces high quality glass products including doors and window panels.
- The factory has three plants. Frames are assembled in Plant A, wood elements are produced in Plant B, and cutting of glass panels and final assembly are done in Plant C.

Example…

- Management has decided to increase production through two additional products: a special type of door and a safety window.
- They consider that there is enough capacity to produce both products without sacrificing the current production, although they might have to compete with the exceeding capacity in Plant C.
- Additionally, no inventories will be allowed, that is, all production will be sold.

Problem Information

Plant	Resources used by unit		Availability of resources
	Product		
	Doors	Windows	
A	1	0	4
B	0	2	12
C	3	2	18
Earnings per unit	3	5	

Example…

The organization is interested in determining the optimal product mix of doors and windows in order to maximize total earnings.

Formulation

- What is the objective?
  - To find how many doors and windows should produce to maximize income.
- Which are the decision variables?
  - The amount of doors (x_1) and windows (x_2) to be produced
- What is the objective function?
  - Total earnings
- What are the constraints?
  - Plant capacities

Standard formulation

Maximize:

$$Z = 3x_1 + 5x_2$$

Subject to:

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$
$$x_1, \quad x_2 \geq 0$$

Solving the problem

- By intuition
- Complete enumeration
- Graphic solution
- Exact mathematical methods
  - Simplex
  - Other approaches
- Heuristics

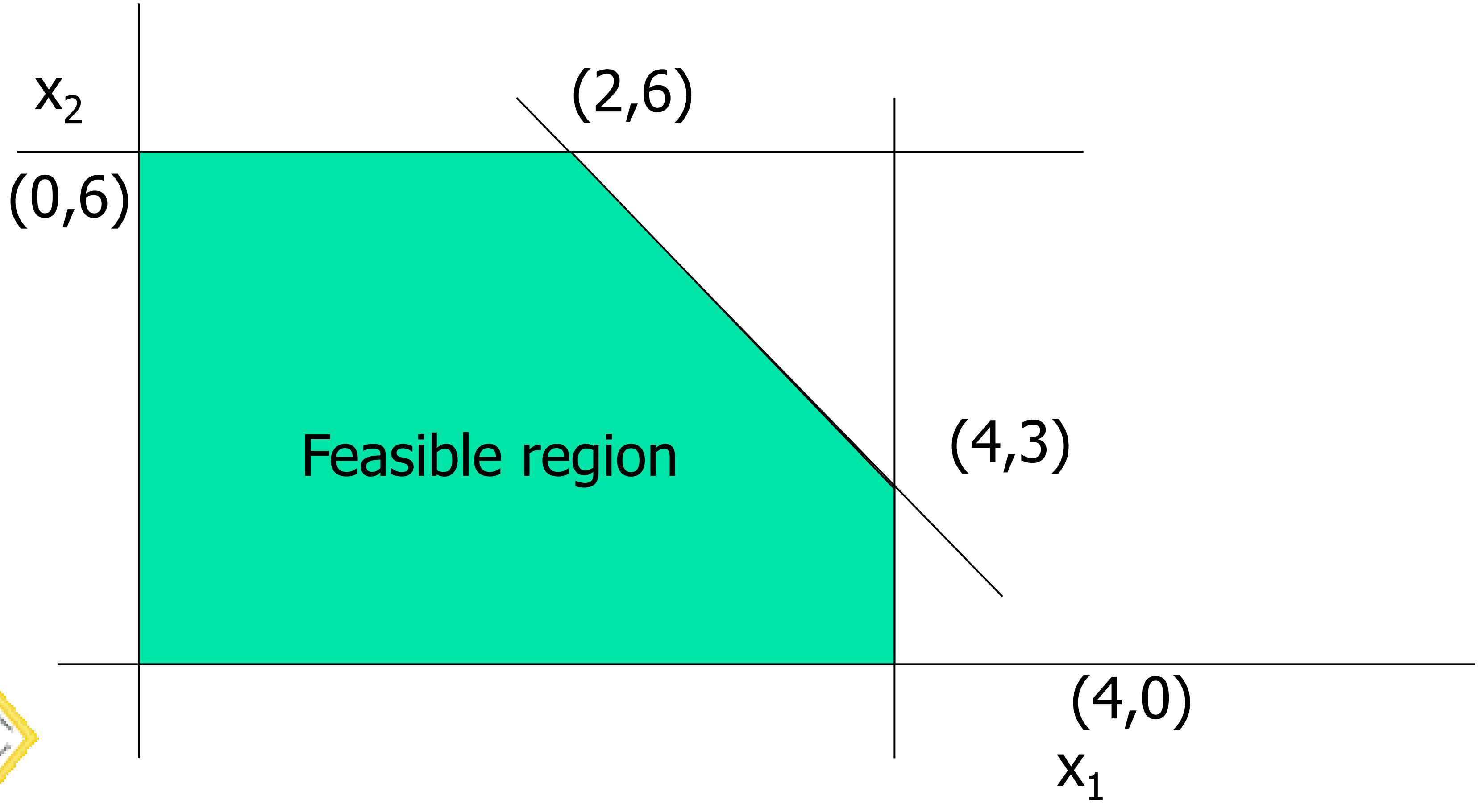
The Graphical Solution

- A LP problem can be represented as a convex region.
- The feasible region is formed by the set of values of the decision variable that simultaneously satisfy all the constraints
- It is a convex region, so that, all the corners are a weighted combination of the points forming the feasible region.
- Candidate solutions for global optima are located in the intersections of the constraints that form the different corners of the convex region.

The optimal solution

- Thus, an optimal solution is located in a corner.
- There is a finite number of corner points.
- If a corner point provides a solution equal or better than any of the adjacent neighbors, then it is optima.

The Graphical Solution of Wyndor



The optimal solution

Corners		Objective function	
		x_1	x_2
x_1	x_2	3	5
0	6	30	
2	6	36	
4	3	27	
4	0	12	

← Optima

Meaning of the solution

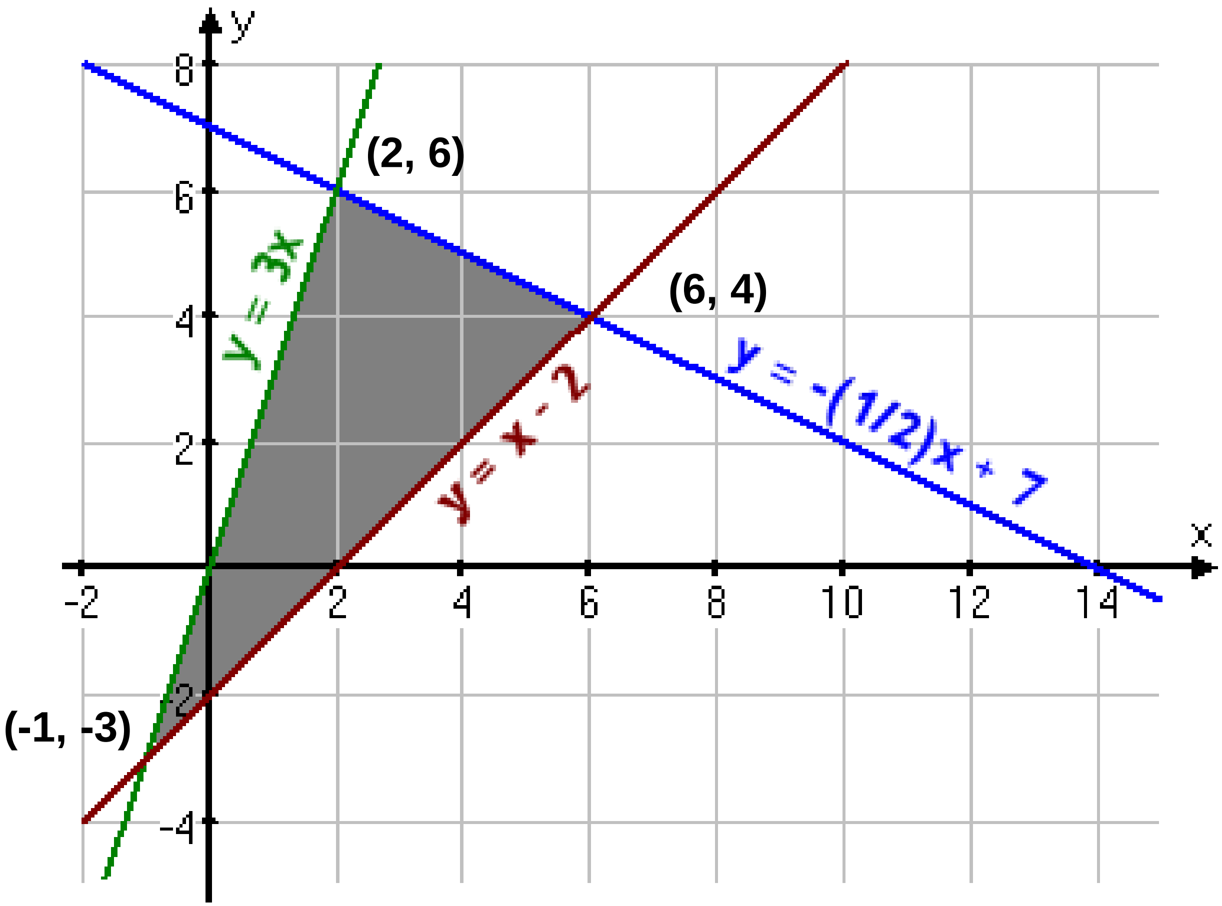
- The optimal production mix is:
  - 2 doors and 6 safety windows
  - A total income of 36 monetary units
  - In Plant A there would be 2 units of resources available
  - No available resources in both Plant B and C ;(2*6 = 12 and 3*2 + 2*6 = 18)

Example

- **Find the maximal and minimal value of $z = 3x + 4y$ subject to the following constraints, for x, and y unrestricted:**

$$\begin{cases} x + 2y \le 14 \\ 3x - y \ge 0 \\ x - y \le 2 \end{cases}$$

$$\begin{Bmatrix} x+2y \le 14 \\ 3x - y \ge 0 \\ x - y \le 2 \end{Bmatrix} \Rightarrow \begin{Bmatrix} y \le -\frac{1}{2}x + 7 \\ y \le 3x \\ y \ge x - 2 \end{Bmatrix}$$



Example 2

- A school is preparing a trip for 400 students. The company who is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only has 9 drivers available. The rental cost for a large bus is $800 and $600 for the small bus. Calculate how many buses of each type should be used for the trip for the least possible cost.

Example 3

- Two Crude Petroleum runs a small refinery on the Texas coast. The refinery distills crude petroleum from two sources, Saudi Arabia and Venezuela, into three main products: gasoline, jet fuel, and lubricants.
- The crudes differ in chemical composition and thus yield different product mixes. Each barrel of Saudi crude yields 0.3 barrel of gasoline, 0.4 barrel of jet fuel, and 0.2 barrel of lubricants. On the other hand, each barrel of Venezuelan crude yields 0.4 barrel of gasoline, but only 0.2 barrel of jet fuel and 0.3 barrel of lubricants. The remaining 10% of each barrel is lost to refining.
- The crudes also differ in cost and availability. Two Crude can purchase up to 9,000 barrels per day from Saudi Arabia at $68 per barrel. Up to 6,000 per day of Venezuelan petroleum are also available at the lower cost of $61 per barrel because of the transportation costs.
- Two Crudes contracts with independent distributors require to produce 2,000 barrels per day of gasoline, 1500 barrels per day of jet fuel, and 500 barrels per day of lubricants. How can these requirements can be fulfilled most efficiently?

Example 4

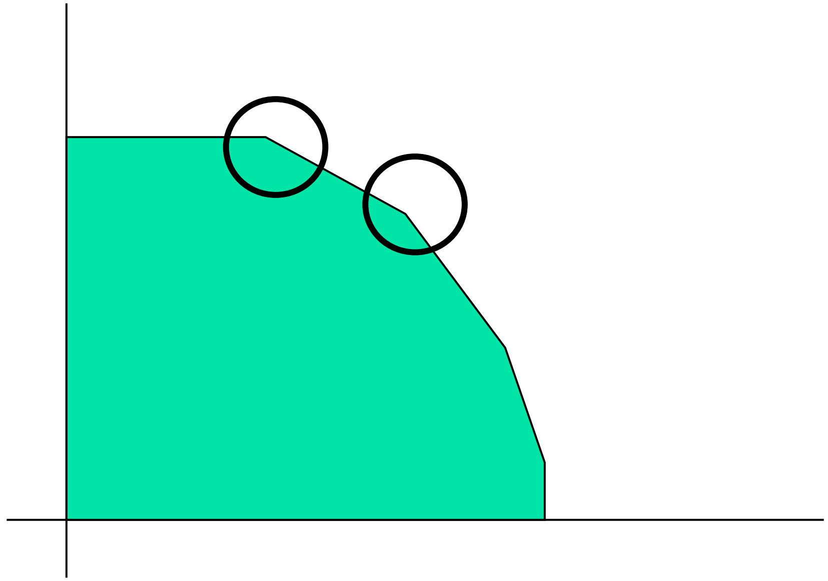
- A transport company has two types of trucks, Type A and Type B. Type A has a refrigerated capacity of 20 m^3 and a non-refrigerated capacity of 40 m^3 while Type B has the same overall volume with equal sections for refrigerated and non-refrigerated stock. A grocer needs to hire trucks for the transport of at least 3,000 m^3 of refrigerated stock and 4 000 m^3 of non-refrigerated stock. The cost per kilometer of a Type A is $30, and $40 for Type B. How many trucks of each type should the grocer rent to achieve the minimum total cost?

Example 5

- A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B.
- At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.
- The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximise the combined sum of the units of X and the units of Y in stock at the end of the week.
- Formulate the problem of deciding how much of each product to make in the current week as a linear program.

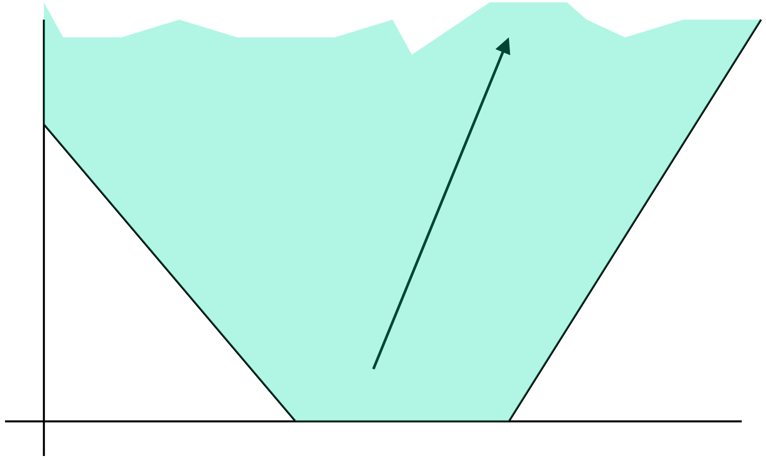
Some special considerations

- Alternate solutions: If there are more than one solution, at least two of them are adjacent.



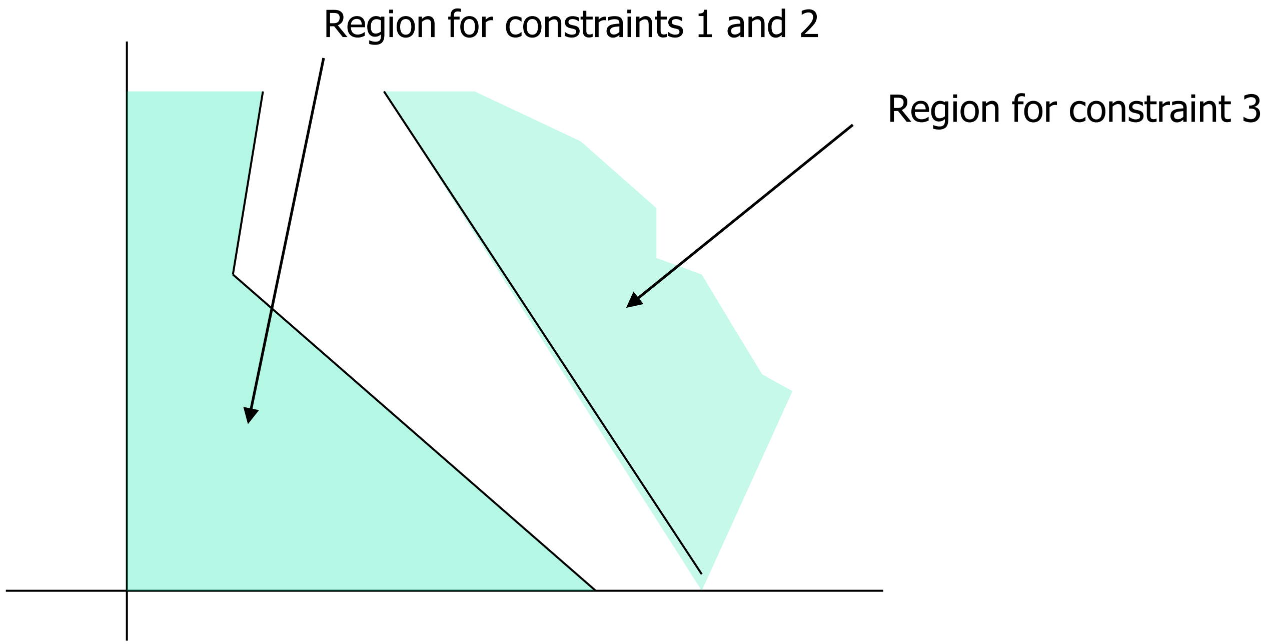
Some special considerations

- Unbounded solution: the region of possible solutions is not bounded by a constraint, thus the solution has infinite possibilities. Normally this situation is due to a formulation error.



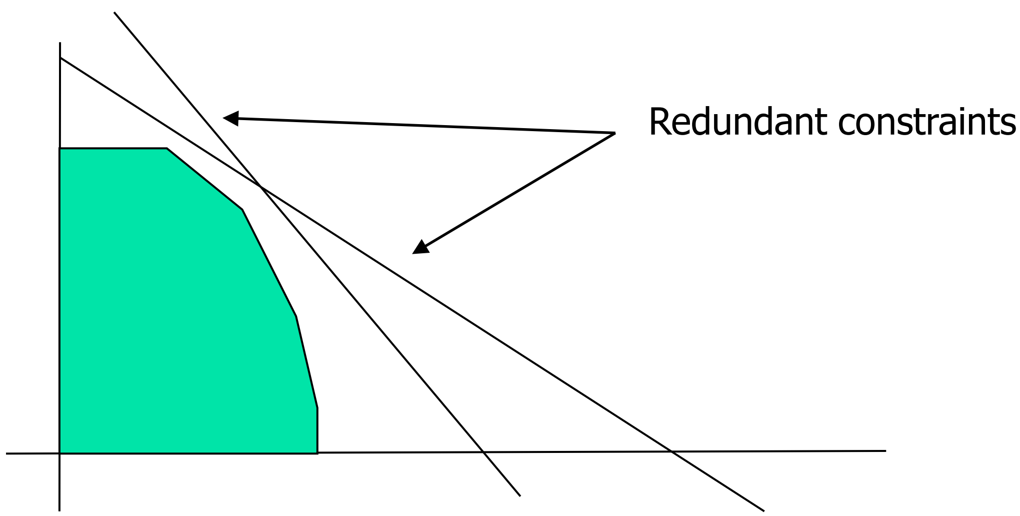
Some special considerations

- Unfeasible solution: when a set of solutions is an empty set, there are no possible points that satisfy all the constraints.



Some special considerations

- Redundant constraints: When there are constraints that do not affect the feasible region, they are redundant in the solution and do not affect it.



The Simplex Method

- Developed in 1947 by George Dantzig as part of a project for the DoD
- Is based on the corner solution property of L. P.
- O(n) complexity

The Simplex Method …

- It is an iterative process
  - Takes advantage of the concept of the corner point.
  - The initial solution requires a standard or augmented formulation.
  - It searches for a solution in all the corner points in $\mathfrak{R}^n$, beginning at the origin of the convex region.
  - It has an optimality test.

General description

Assume a standard LP formulation:

$$
\begin{aligned}
&\text{Max (o Min)} \qquad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \\
&\text{s.t.} \\
&\qquad \begin{array}{rcl}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &=& b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &=& b_2 \\
\vdots &=& \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &=& b_m \\
x_i &\geq& 0 \quad \forall\, i = 1 \ldots n
\end{array}
\end{aligned}
$$

Such that:

$$
[A] = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix} \qquad \{x\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \qquad \{b\} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{Bmatrix}
$$

$$
\begin{aligned}
&\max \qquad x_0 = \mathbf{c}^T\mathbf{x} \\
&\text{subject to} \begin{cases} A\mathbf{x} \leq \mathbf{b}, \\ \mathbf{x} \geq \mathbf{0}. \end{cases}
\end{aligned}
$$

In the canonical form $[A]\{x\} = \{b\}$

Augmented or standard formulation: to create equalities from inequalities

- The case of constraints type ≤
  - It is necessary to add an slack variable

    $x_1 \leq 4; x_1 = 4 - x_3; x_1 + x_3 = 4$
- The case of constraints type ≥
  - It is necessary to add a surplus variable such that

    $x_1 \geq 5; x_1 = 5 + x_4; x_1 - x_4 = 5$
  - It is necessary to add and artificial variable x_5 such that $x_1 - x_4 + x_5 = 5$ and does not violates the non negative constraint $x_j > 0$ in the initial solution.
  - The coefficient in the objective function will be +/- M>>0 such that x_5 has an initial solution of zero
  - The case of constraints type =
  - An artificial variable is added with +/- M >> 0 as a coefficient in the objective function such that:

    $x_1 = 5; x_1 + x_6 = 5$

The initial solution

- Simplex assumes an initial solution at the origin, thus all the initial variables are set in zero.
- Since this condition violates the main constraints in the formulation, Simplex needs to generate an augmented formulation.

The augmented solution

- It is the solution of a linear programming problem originally formulated in the standard manner
- It is an augmented corner point solution
- A basic feasible solution is a feasible augmented corner point solution

Properties of a solution

- Degrees of freedom: it is the difference between the number of variables, including the slack, surplus or artificial variables and the number of constraints, not including the nonnegative.
- To solve the system it is necessary to assume arbitrary values, zero in this case.
- The variables that are set to zero are known as non basic variables.
- The variables included in the solution are known as basic variables.

The initial solution

The standard formulation: $$[A \quad I]\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_s\end{bmatrix} = A\mathbf{x} + I\mathbf{x}_s = \mathbf{b}.$$

The initial tableau

Slack variables ($x_{n+1}, \dots, x_{n+m}$)

	x_1	x_2	$\cdots$	x_s	$\cdots$	x_n	x_{n+1}	$\cdots$	x_{n+r}	$\cdots$	x_{n+m}	b
x_{n+1}	a_{11}	a_{12}	$\cdots$	a_{1s}	$\cdots$	a_{1n}	1	$\cdots$	0	$\cdots$	0	b_1
x_{n+2}	a_{21}	a_{22}	$\cdots$	a_{2s}	$\cdots$	a_{2n}	0	$\cdots$	0	$\cdots$	0	b_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_{n+r}	a_{r1}	a_{r2}	$\cdots$	a_{rs}	$\cdots$	a_{rn}	0	$\cdots$	1	$\cdots$	0	b_r
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_{n+m}	a_{m1}	a_{m2}	$\cdots$	a_{ms}	$\cdots$	a_{mn}	0	$\cdots$	0	$\cdots$	1	b_r
x_0	$-c_1$	$-c_2$	$\cdots$	$-c_s$	$\cdots$	$-c_n$	0	$\cdots$	0	$\cdots$	0	0

At the initial solution the decision variables $x_1, \dots, x_n = 0$, and are non basic variables in the solution

The set of variables in the solution are called basic variables, and the solution a basic feasible solution

The iterative process

- In order to find a better adjacent solution, basic variable will become non basic and a non basic will enter the solution as a basic.
- The entering variable will be the one that improves the objective solution faster.
- The leaving variable will be the first one to become zero.
- The optimal solution is found when there are no more improving non basic variables.

Moving within the $\Re^n$ space

- Let β the set of basic variables, such that in the initial solution $\beta = \{x_{n+i}\}_{i=1, m}$
- Let η be the set of non basic variables, such that in the initial solution $\eta=\{x_i\}_{i=1,n}$
- To replace $x_r \in \beta$ by $x_s \in \eta$ the a_{rs} element is called the pivot point and the operation becomes a Gaussian elimination such that:

	j	s
i	a_{ij}	a_{is}
r	a_{rj}	a^*_{rs}

becomes

	j	s
i	$a_{ij} - a_{rj}a_{is}/a_{rs}$	0
r	a_{rj}/a_{rs}	1

Moving within the $\Re^n$ space

- The entering x_s will be selected according to an optimality test, i. e., the most positive or negative variable.
- One strategy would be to select whichever variable has the greatest reduced cost . In linear programming, reduced cost, or opportunity cost, is the amount by which an objective function coefficient would have to improve (so increase for maximization problem, decrease for minimization problem) before it would be possible for a corresponding variable to assume a positive value in the optimal solution.
- The leaving x_r must be selected as the basic variable corresponding to the smallest positive ration of the values of the current right hand side of the current positive constraint coefficient of the entering non-basic variable x_s

$$\frac{y_{r0}}{y_{rs}} = \min_i \{ \frac{y_{i0}}{y_{is}} \mid y_{is} > 0 \}$$

The Wyndor case: the standard formulation

Minimize Z such that:

$$Z - 3x_1 - 5x_2 = 0$$

s.t.

$$\begin{aligned} x_1 + x_3 &= 4 \\ 2x_2 + x_4 &= 12 \\ 3x_1 + 2x_2 + x_5 &= 18 \end{aligned}$$

- There are to decision variables and three slack variables. In addition there are three constraints. Thus, the degree of freedom is two.

The initial solution

- The basic initial solution will be:

  $x_1 = x_2 = 0$ and

  $x_3 = 4$

  $x_4 = 12$

  $x_5 = 18$

- Since this solution is not an optima, the iteration process begins.

The Wyndor case: the standard formulation – initial tableau

	X_1	x_2	x_3	x_4	x_5	b	
x_3	1	0	1	0	0	4	4/0
x_4	0	2	0	1	0	12	12/2
X_5	3	2	0	0	1	18	18/2
z	-3	-5	0	0	0	0	

Enters

Leaves

Formulation and solution of the Wyndor Example with AMPL

wyndor1.mod: Bloc de notas

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```
var x1 >=0;
var x2 >=0;
maximize z: 3*x1+5*x2;
subject to
PlantA: x1<=4;
PlantB:2*x2<=12;
PlantC: 3*x1+2*x2<=18;
solve;
display z, x1,x2;
```

sw: running ampl

File Edit Help

```
sw: ampl
ampl: option solver cplex;
ampl: model f:\models\wyndor1.mod;
CPLEX 12.6.0.0: optimal solution; objective 36
1 dual simplex iterations (0 in phase I)
z = 36
x1 = 2
x2 = 6

ampl:
```

example2.mod: Bloc de no

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```
var x;
var y;
maximize z: 3*x+4*y;
subject to
c1:x + 2*y <=4;
c2:3*x - y>=0;
c3:x - y <=2;
option solver cplex;
solve;
display z, x, y;
```

sw: running ampl

File Edit Help

```
ampl: reset;
ampl: model f:\models\example2.mod;
CPLEX 12.6.0.0: optimal solution; objective 10.66666667
2 dual simplex iterations (2 in phase I)
z = 10.6667
x = 2.66667
y = 0.666667

ampl: reset;
ampl: model f:\models\example3.mod;
CPLEX 12.6.0.0: optimal solution; objective -15
0 dual simplex iterations (0 in phase I)
z = -15
x = -1
y = -3

ampl:
```

.mod: Bloc de notas

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```
var x;
var y;
minimize z: 3*x+4*y;
subject to
c1:x + 2*y <=4;
c2:3*x - y>=0;
c3:x - y <=2;
option solver cplex;
solve;
display z, x, y;
```

Formulation and Solution of the sign unrestricted example

The Duality

- Every linear programming problem, referred to as a primal problem, can be converted into a dual problem, which provides an upper bound to the optimal value of the primal problem
- The primary problem and the dual problem are complementary. A solution to either one determines a solution to both.
- In the primal problem, the objective function is a linear combination of *n* variables. There are *m* constraints, each of which places an upper bound on a linear combination of the *n* variables. The goal is to maximize the value of the objective function subject to the constraints. A solution is a vector (a list) of *n* values that achieves the maximum value for the objective function.
- In the dual problem, the objective function is a linear combination of the *m* values that are the limits in the *m* constraints from the primal problem. There are *n* dual constraints, each of which places a lower bound on a linear combination of *m* dual variables.

The Dual

- Every maximization (minimization) problem in L. P. has an equivalent dual minimization (maximization) problem.

Primal

$$\text{Max } Z = \sum_{j=1}^{n} c_j x_j$$

s. t. :

$$\sum_{j=1}^{n} a_{i,j} x_j \leq b_i \ \forall\ i = 1, 2, \ldots, m$$

$$x_j \geq 0$$

Dual:

$$\text{Min } Y = \sum_{i=1}^{m} b_i y_i$$

s. t. :

$$\sum_{i=1}^{m} a_{i,j} y_i \geq cj \ \forall\ j = 1, 2, \ldots, n$$

$$y_i \geq 0$$

Relationship Primal - Dual

		Primal problem		
Dual Problem	Coefficients of y_i	Coefficients of $x_1 \quad x_2 \ldots x_n$	$\leq b_i$	Coefficients of the Objective function (Minimize)
	Y_1	$a_{1,1} \; a_{1,2} \ldots a_{1,n}$	b_1	
	y_2	$a_{2,1} \; a_{2,2} \ldots a_{2,n}$	b_2	
	. . .	. . .	. . .	
	y_m	$a_{m,1} \; a_{m,2} \ldots a_{m,n}$	b_m	
	$\geq c_j$	$c_1 \quad c_2 \ldots c_n$		

18:47:55		Monday	September	13	2010		
Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1 X1	2.0000	3.0000	6.0000	0	basic	0	7.5000
2 X2	6.0000	5.0000	30.0000	0	basic	2.0000	M
Objective	Function	(Max.) =	36.0000				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1 C1	2.0000	<=	4.0000	2.0000	0	2.0000	M
2 C2	12.0000	<=	12.0000	0	1.5000	6.0000	18.0000
3 C3	18.0000	<=	18.0000	0	1.0000	12.0000	24.0000

Primal solution

18:49:19		Monday	September	13	2010		
Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1 C1	0	4.0000	0	2.0000	at bound	2.0000	M
2 C2	1.5000	12.0000	18.0000	0	basic	6.0000	18.0000
3 C3	1.0000	18.0000	18.0000	0	basic	12.0000	24.0000
Objective	Function	(Min.) =	36.0000				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1 X1	3.0000	>=	3.0000	0	2.0000	0	7.5000
2 X2	5.0000	>=	5.0000	0	6.0000	2.0000	M

Dual solution

The solution of the dual

- The solution of $\{y_j\}j_{i=1,m}$ represents the contribution of the unit profit of resource j when the primal is solved.
- The shadow price is the change in the objective value of the optimal solution of an optimization problem obtained by relaxing the constraint by one unit – it is the marginal utility of relaxing the constraint, or equivalently the marginal cost of strengthening the constraint.
- Thus the solution of the dual defines the shadow prices of the resources.

Post-optimal or sensitivity analysis

- It is one of the most important steps in LP
- It consists of determining how sensible is the model's optimal solution if certain parameters such as the Objective Function coefficients ot the independent terms of the los coeficientes deconstraints change.

Post-optimal or Sensitivity analysis

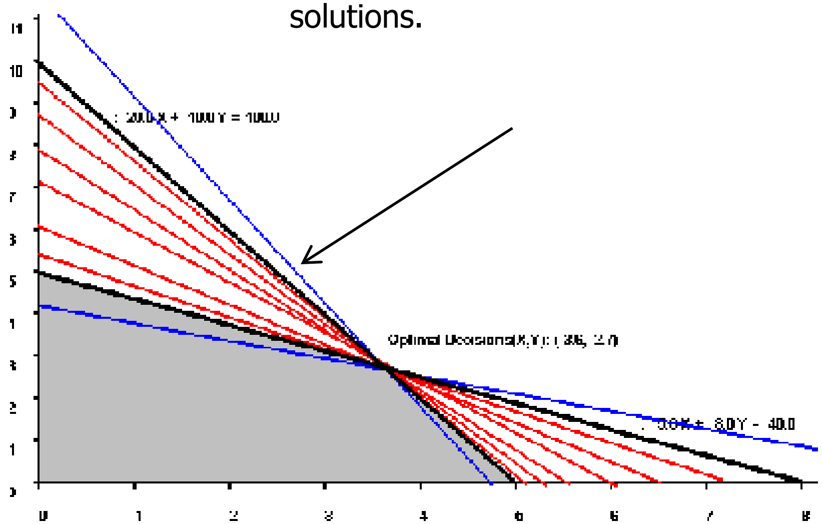
- Studies the possibility of variations of the solution if different parameters vary.
- It is used to determine the variation of a coefficient without varying the solution.
  - **Changes in the coefficients of a non basic variable:** do not affect the solution since they are not part of the solution.
  - **Introduction of a new variable:** An analysis of the results of adding a new constraint in the dual.
  - **Changes in b_j**: they may change the problem and the shadow prices.
  - **Changes in the coefficients of the basic variables:** they affect the value of the objective function.

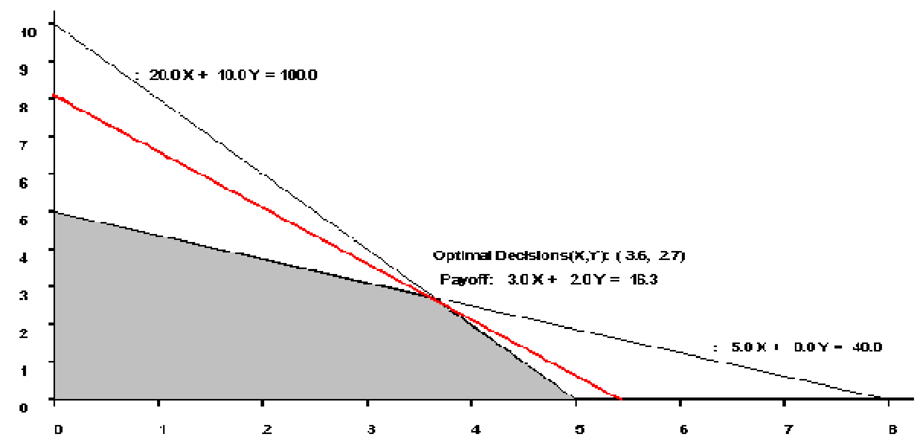
Analysis for the Objective Function coefficients

- The objective is to find the range of values that keep the original solution optimal

All the red lines keep the solution optimal. The blue lines generate new optimal solutions.

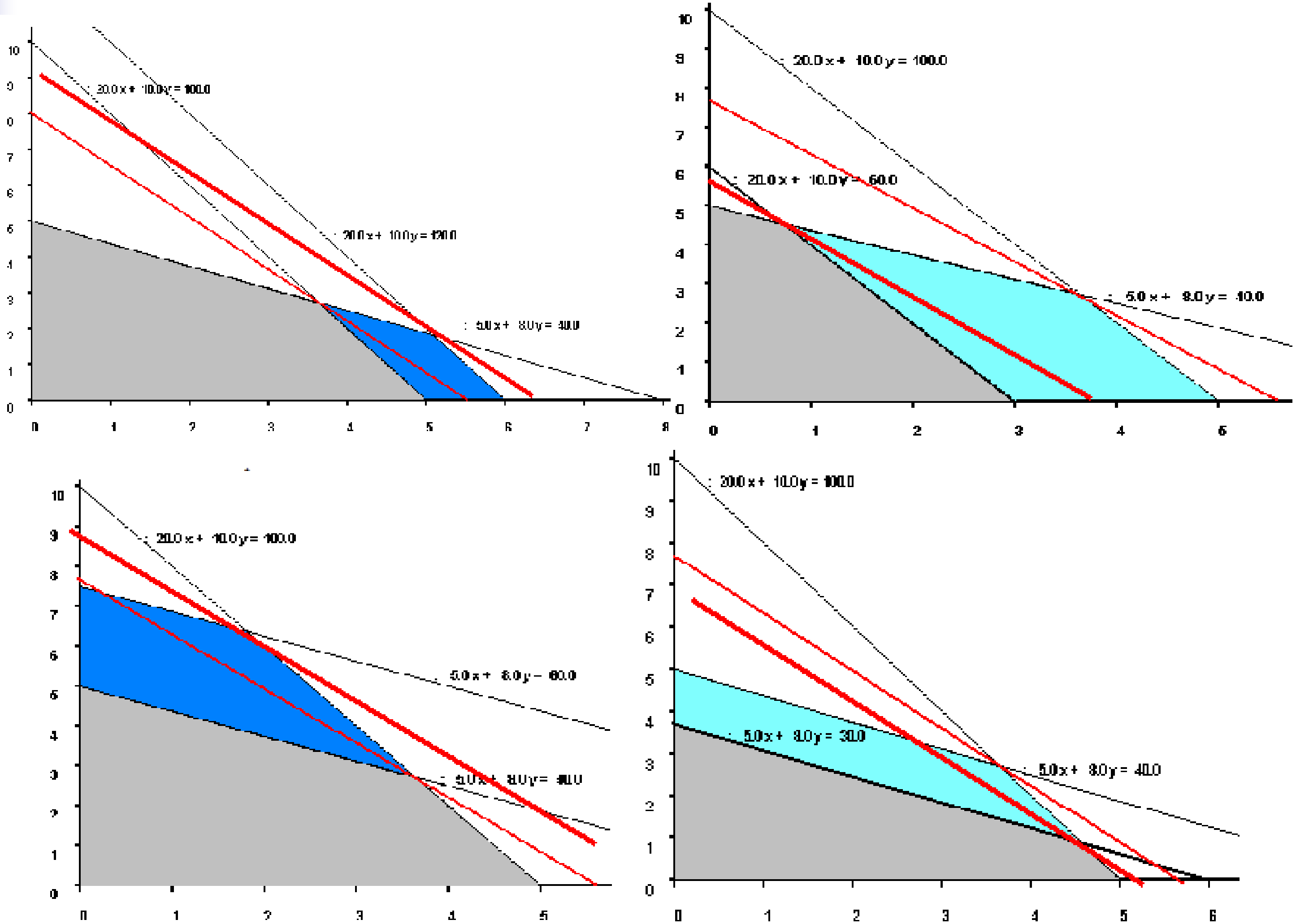
$$
\begin{aligned}
Máx \quad & Z = 3x + 2y \\
s/a \quad & 5x + 8y \leq 40 \\
& 20x + 10y \leq 100 \\
& x \geq 0 ; y \geq 0
\end{aligned}
$$



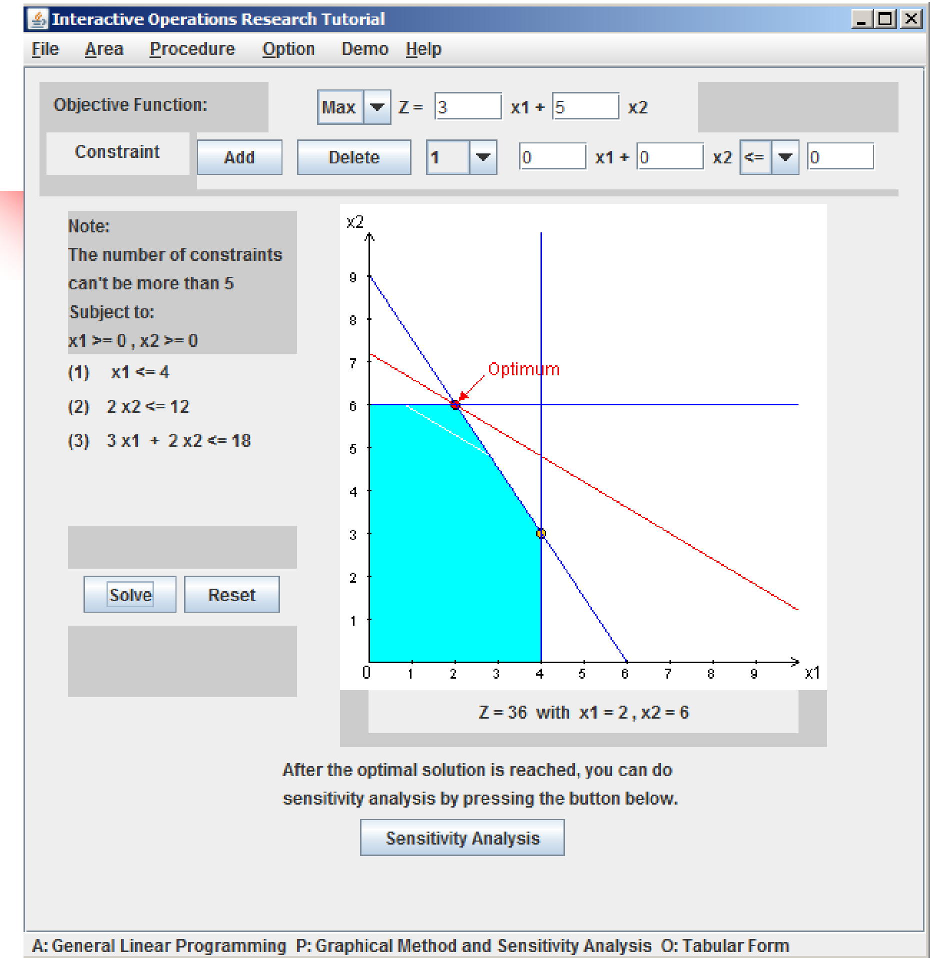


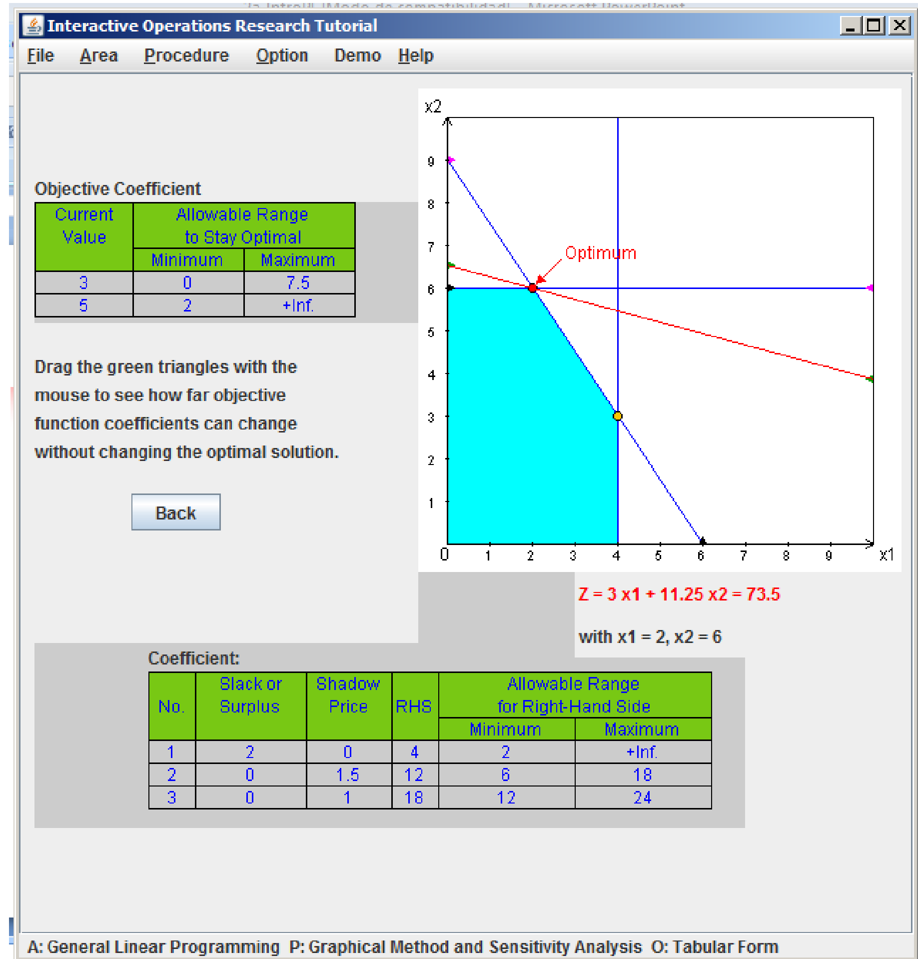
Analysis for independent terms

- The objective is to keep the original dual solution.

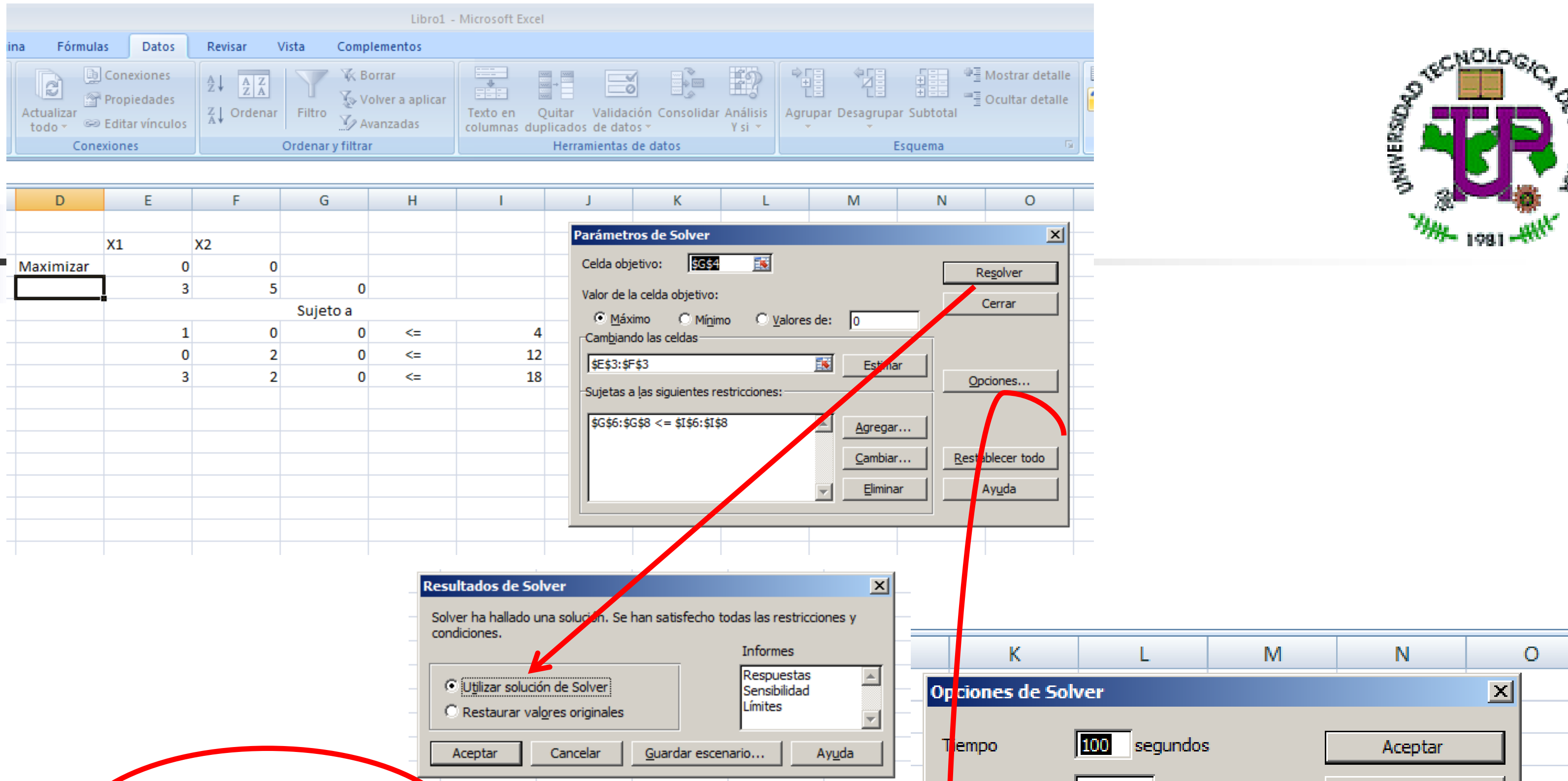


IOR Tutorial





Solver



	X1	X2			
Maximizar	2	6			
	3	5	36		
			Sujeto a		
	1	0	2	<=	4
	0	2	12	<=	12
	3	2	18	<=	18

Variable -->	X1	X2	Direction	R. H. S.
Minimize	800	600		
C1	1	1	<=	9
C2	50	40	>=	400
C3	1		<=	10
C4		1	<=	8
LowerBound	0	0		
UpperBound	M	M		
VariableType	Continuous	Continuous		

Variable -->	C1	C2	C3	C4	Direction	R. H. S.
Maximize	9	400	10	8		
X1	1	50	1		<=	800
X2	1	40		1	<=	600
LowerBound	-M	0	-M	-M		
UpperBound	0	M	0	0		
VariableType	Continuous	Continuous	Continuous	Continuous		

	09:12:50		Monday	September	10	2012		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	4.0000	800.0000	3,200.0000	0	basic	750.0000	M
2	X2	5.0000	600.0000	3,000.0000	0	basic	-M	640.0000
	Objective	Function	(Min.) =	6,200.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	9.0000	<=	9.0000	0	-200.0000	8.0000	9.6000
2	C2	400.0000	>=	400.0000	0	20.0000	370.0000	450.0000
3	C3	4.0000	<=	10.0000	6.0000	0	4.0000	M
4	C4	5.0000	<=	8.0000	3.0000	0	5.0000	M

	09:13:35		Monday	September	10	2012		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	C1	-200.0000	9.0000	-1,800.0000	0	basic	-9.6000	-8.0000
2	C2	20.0000	400.0000	8,000.0000	0	basic	370.0000	450.0000
3	C3	0	10.0000	0	-6.0000	at bound	-M	-4.0000
4	C4	0	8.0000	0	-3.0000	at bound	-M	-5.0000
	Objective	Function	(Max.) =	6,200.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	X1	800.0000	<=	800.0000	0	4.0000	750.0000	M
2	X2	600.0000	<=	600.0000	0	5.0000	-M	640.0000

Example

- A building supply has two locations in town. The office receives orders from two customers, each requiring 3/4-inch plywood. Customer A needs fifty sheets and Customer B needs seventy sheets. The warehouse on the east side of town has eighty sheets in stock; the west-side warehouse has forty-five sheets in stock. Delivery costs per sheet are as follows: $0.50 from the eastern warehouse to Customer A, $0.60 from the eastern warehouse to Customer B, $0.40 from the western warehouse to Customer A, and $0.55 from the western warehouse to Customer B.
- Find the shipping arrangement which minimizes costs.

- **Production planning problem**
- A company manufactures four variants of the same product and in the final part of the manufacturing process there are assembly, polishing and packing operations. For each variant the time required for these operations is shown below (in minutes) as is the profit per unit sold.

	Assembly	Polish	Pack	Profit (£)
Variant 1	2	3	2	1.50
2	4	2	3	2.50
3	3	3	2	3.00
4	7	4	5	4.50

- Given the current state of the labor force the company estimate that, each year, they have 100000 minutes of assembly time, 50000 minutes of polishing time and 60000 minutes of packing time available. How many of each variant should the company make per year and what is the associated profit?
- Suppose now that the company is free to decide how much time to devote to each of the three operations (assembly, polishing and packing) within the total allowable time of 210000 (= 100000 + 50000 + 60000) minutes. How many of each variant should the company make per year and what is the associated profit?

Some practice on MPL

- Formulate and solve, using MPL all the examples previously seen.