Computational compexity

H. R. Alvarez A., Ph. D.

¿What is it ?

- Part of the computational theory that studies the resources required during computation to solve a problem. The commonly studied resources are:
  - time (number of execution steps of an algorithm for solving a problem)
  - space (amount of memory used to solve a problem).

Combinatory complexity

- It is based on the number of components of a system, or the number of possible combinations to be performed when making a decision.
- It is a function of both the variables and the functions that govern or shape the system

Algorithms and problem solving

- From the Greek and Latin, “dixit algorithmus”, oritinally from Persian mathematician Al-Khwarizmi
- Informally, an algorithm is a well-defined computational procedure that takes a set of values (inputs) and produces a set of values (outputs) using a sequence of computational steps in the transformation.
- Prescribed set of well-defined, finite and ordered rules or instructions, that enables a solution process through successive steps that generate no doubt who should perform this activity.

Types of algorithms

- Ordering algorithms:
  - Let the input be a sequence of *n* numbers $(a_1, a_2,\ldots, a_n)$
  - The output will be the permutation or ordering $(a_1', a_2',\ldots, a_n')$, such that $a_1' \leq a_2' \leq,\ldots, \leq a_n'$
- Search algorithms:
  - Let the input be a sequence of *n* numbers $(a_1, a_2,\ldots, a_n)$
  - The output will be a number a^k such that $a^k \supset$ {characteristics}

Design and solution techniques

- **Greedy algorithms:** select the most promising elements of the set of candidates to find a solution. In most cases the solution is not optimal.
- **Parallel algorithms:** allow dividing a problem into sub problems so that they can run simultaneously on multiple processors.
- **Probabilistic algorithms**: some of the steps of such algorithms are based on pseudo-random values.
- **Deterministic algorithms**: the behavior of the algorithm is sequential: each step of the algorithm has only one preceding step and another successor step.

Design and solution techniques

- **Non-deterministic algorithms**: the behavior of the algorithm is a tree and each step of the algorithm can branch to any number of immediately following steps, plus all the branches are executed simultaneously.
- **Divide and conquer**: divides the problem into disjoint subsets obtaining a solution for each subset. It then unites them, achieving a solution to the whole problem
- **Meta heuristics**: It finds suboptimal or approximate solutions to problems based on prior knowledge (sometimes called experience).

Design and solution techniques

- **Dynamic programming**: tries to solve a problem through different sequential steps, tracking back possible solutions. will examine the previously solved subproblems and will combine their solutions to give the best solution for the given problem.
- **Branch and bound:** Based on the construction of the solutions to a problem through an implicit tree that runs in a controlled manner by finding the best solutions.

Properties (no for paralell algorithms)

- **Sequential time**. An algorithm runs in discretized -step by step time, thus defining a sequence of "computational" states for each valid entry.
- **Abstract state**. Each computational state can be formally described using a first-order structure and each algorithm is independent of its implementation
- **Bounded exploration**. The transition from one state to the next is completely determined by a fixed and finite description; that is, between each state and the next you can only take into account a fixed and limited amount of possible current states

The problem

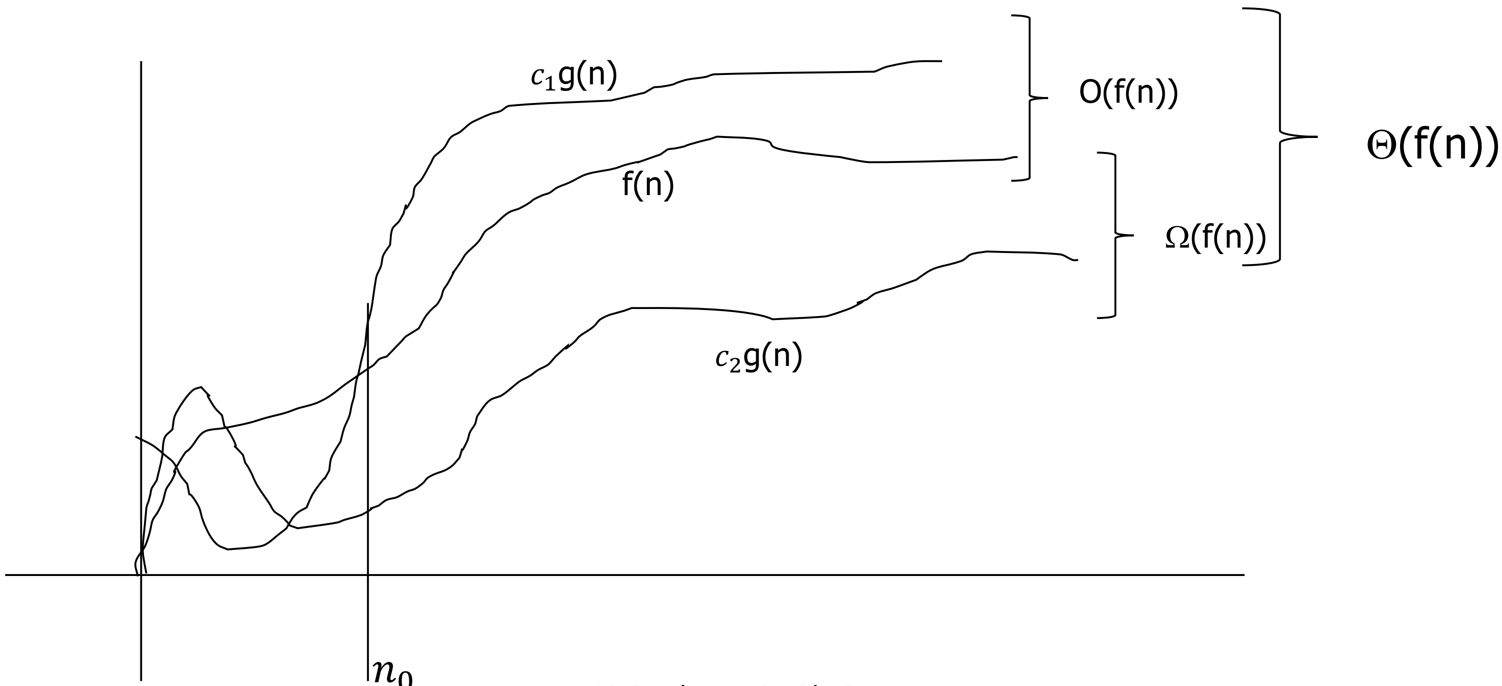
- The resulting problem of a mathematical model has three elements:
  - The problem: the ultimate question
  - Elements: a list of parameters, variables and relationships, characteristic of the solution
  - Instances: parameter values

Type of problems

- Tractables or decidable problems: there are algorithms capable of optimally solving them.
- Undecidable or not tractables problems: there are no algorithms that can optimally solve them.

Efficiency of an algorithms

- The notation that describes the behavior, as a function of time, of the excecution of an algorithms, is asymptotically approximate.
- Θ(f(n)) ={f(n) $\exists$ c_1, c_2, n_0 / 0≤ c_1g(n) ≤ f(n) ≤ c_2g(n) $\forall$ n $\geq n_0$}
- O(f(n)) = {f(n) $\exists$ c_1, n_0 / 0≤c_1g(n) ≤ f(n) $\forall$ n $\geq n_0$}
- Ω)f(n)) = {f(n) $\exists$ c_2, n_0 / 0≤ f(n) ≤ c_2g(n) $\forall$ n $\geq n_0$}



Polynomial problems

- One problem is Polynomial (P) if there is a deterministic polynomial time algorithm to solve it.
  - When the running time of an algorithm is less than a certain value determined in terms of he length of the input variable (n) a problem can be solved in polynomial time.
- An algorithm is efficient if a problem can be solved such that the number of steps to resolve grows polynomially depending on their size.
- Can be approximated to a solution in terms of n^k

Non-Polynomial (NP) Problem

- If there is no deterministic polynomial algorithm to solve it.
- A special case are the intractable problems, which include:
  - Consistently intractable: Those that are so difficult that not even a non polynomial time algorithm can solve it.
  - Seemingly intractable: The problem is so difficult that an exponential time is required to find a solution. The solution is so large that can not be expressed as a polynomial function of the input.
- Within the class NP "difficult" NP-complete problems as defined If there is no deterministic polynomial algorithm to solve.

Efficiency of some algorithms

- Simplex $O(n^k)$
- Interior point (Karmakar and others) $O(n\log(n))$
- Integer programing NP-Complete
  - Branch and bound $O(k^n)$
- TSP NP-Complete