① $dS = r^2 \operatorname{sen}\theta \, d\theta \, d\phi$

$$S = \int_{\phi=0}^{2\pi} \int_{\theta=\frac{\pi}{4}}^{\frac{2\pi}{3}} r^2 \operatorname{sen}\theta \, d\theta \, d\phi$$

$$= \int_{\phi=0}^{2\pi} r^2 \left(-\cos\theta\right)\Big]_{\theta=\frac{\pi}{4}}^{\frac{2\pi}{3}} d\phi$$

$$= 10^2 \left[-\cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\right] \int_{\phi=0}^{2\pi} d\phi$$

$$= 10^2 \left[0.5 + \frac{1}{\sqrt{2}}\right] \phi \Big|_{\phi=0}^{2\pi}$$

$$= 10^2 \left[0.5 + \frac{1}{\sqrt{2}}\right] 2\pi$$

$= 758.45$ unidades^2

② $dv = \rho \, d\rho \, d\phi \, dz$

$$V = \int_{z=-1}^{4} \int_{\phi=\frac{\pi}{3}}^{\pi} \int_{\rho=2}^{5} \rho \, d\rho \, d\phi \, dz$$

$$= \int_{z=-1}^{4} \int_{\phi=\frac{\pi}{3}}^{\pi} \left. \frac{\rho^2}{2} \right|_{\rho=2}^{5} d\phi \, dz$$

$$= \int_{z=-1}^{4} \int_{\phi=\frac{\pi}{3}}^{\pi} \frac{25}{2} - \frac{4}{2} \, d\phi \, dz$$

$$= \int_{z=-1}^{4} \left. \frac{21}{2} \phi \right|_{\phi=\frac{\pi}{3}}^{\pi} dz$$

$$= \int_{z=-1}^{4} \frac{21\pi}{2} - \frac{21\pi}{6} \, dz$$

$$= \left. 7\pi z \right|_{z=-1}^{4}$$

$$= 7\pi(4+1)$$

$$= 35\pi$$

$$= 109.95$$

③ a) $$E = \frac{Q_1 (r - r_1)}{4\pi\epsilon_0 |r - r_1|^3} + \frac{Q_2 (r - r_2)}{4\pi\epsilon_0 |r - r_2|^3}$$

$$= \frac{Q_1[(5,0,6)-(4,0,-3)]}{4\pi\epsilon_0 |(5,0,6)-(4,0,-3)|^3} + \frac{4\times10^{-9}[(5,0,6)-(2,0,1)]}{4\pi\epsilon_0 |(5,0,6)-(2,0,1)|^3}$$

$$= \frac{Q_1 (1,0,9)}{4\pi\epsilon_0 (\sqrt{82})^3} + \frac{4\times10^{-9}(3,0,5)}{4\pi\epsilon_0 (\sqrt{34})^3}$$

$$E_z = 0$$

$$\frac{9 Q_1}{4\pi\epsilon_0 (\sqrt{82})^3} + \frac{4\times10^{-9}(5)}{4\pi\epsilon_0 (\sqrt{34})^3} = 0$$

$$\frac{9 Q_1}{\cancel{4\pi\epsilon_0}(\sqrt{82})^3} = -\frac{4\times10^{-9}(5)}{\cancel{4\pi\epsilon_0}(\sqrt{34})^3}$$

$$Q_1 = -\frac{4\times10^{-9}(5)(\sqrt{82})^3}{9(\sqrt{34})^3}$$

$$Q_1 = -8.3232\times10^{-9}\,C$$

$$Q_1 = -8.3232\,mC$$

b) $\mathbf{F}(5,0,6) = q\,\mathbf{E}(5,0,6)$

$F_x = 0$

$$\frac{q Q_1}{4\pi\epsilon_0(\sqrt{32})^3} + \frac{3 q Q_2}{4\pi\epsilon_0(\sqrt{34})^3} = 0$$

$$\frac{\cancel{q} Q_1}{\cancel{4\pi\epsilon_0}(\sqrt{82})^3} = -\frac{3\cancel{q} Q_2}{\cancel{4\pi\epsilon_0}(\sqrt{34})^3}$$

$$Q_1 = -\frac{3(4\times10^{-9}\,C)(\sqrt{82})^3}{(\sqrt{34})^3}$$

$$Q_1 = -44.95\times10^{-9}\,C$$

$$Q_1 = -44.95\,nC$$

(4) $\mathbf{f}(x,y,z) = x\,\mathbf{a}_x + y\,\mathbf{a}_y + z\,\mathbf{a}_z$ en $\mathbb{R}^3$

$$\|\mathbf{f}(x,y,z)\|^2 = \mathbf{f}\cdot\mathbf{f} = x^2 + y^2 + z^2$$

a) $$\nabla\|\mathbf{f}\|^2 = \frac{\partial}{\partial x}(x^2)\,\mathbf{a}_x + \frac{\partial}{\partial y}(y^2)\,\mathbf{a}_y + \frac{\partial}{\partial z}(z^2)\,\mathbf{a}_z$$

$$= 2x\,\mathbf{a}_x + 2y\,\mathbf{a}_y + 2z\,\mathbf{a}_z$$

b) $$\nabla\cdot\mathbf{f} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$$

$$= 1 + 1 + 1$$

$$= 3$$

c) $$\nabla\times\mathbf{f} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(z) - \frac{\partial}{\partial z}(y)\right]\mathbf{a}_x + \left[\frac{\partial}{\partial z}(x) - \frac{\partial}{\partial x}(z)\right]\mathbf{a}_y$$

$$\left[\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right]\mathbf{a}_z$$

$$= 0$$

d) $$\nabla^2\|\mathbf{f}\|^2 = \frac{\partial^2(x^2)}{\partial x^2} + \frac{\partial^2(y^2)}{\partial y^2} + \frac{\partial(z^2)}{\partial z^2}$$

$$= 2 + 2 + 2$$

$$= 6$$

e) $\|f\|^2 = x^2 + y^2 + z^2 \rightarrow$ COORDENADAS CARTESIANAS

$\|f\|^2 = r^2 \rightarrow$ COORDENADAS ESFÉRICAS

$\therefore$ GRADIENTE

$$\nabla f = \frac{\partial}{\partial r}(r^2)\, a_r$$

$$= 2r\, a_r$$

$$= 2r \frac{\mathbf{r}}{\|\mathbf{r}\|}$$

$$= 2r \frac{\mathbf{r}}{r}$$

$$= 2\mathbf{r}$$

LAPLACIANO

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}(r^2)\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 (2r)\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[2r^3\right]$$

$$= \frac{1}{\cancel{r^2}}\left[6\cancel{r^2}\right]$$

$$= 6$$

5

$$\rho_v = \nabla \cdot \mathbf{D}$$

a) $$\rho_v = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot \rho \,\text{sen}\,\phi) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(2\rho \cos\phi) + \frac{\partial}{\partial z}(2z^2)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^2 \,\text{sen}\,\phi) + \frac{1}{\cancel{\rho}} \cdot 2\cancel{\rho}\frac{\partial}{\partial \phi}(\cos\phi) + \frac{\partial}{\partial z}(2z^2)$$

$$= \frac{1}{\cancel{\rho}}[2\cancel{\rho}\,\text{sen}\,\phi] + 2(-\text{sen}\,\phi) + 4z$$

$$= \cancel{2\,\text{sen}\,\phi} - \cancel{2\,\text{sen}\,\phi} + 4z$$

$$= 4z$$

b) $$\rho_v = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\left(\frac{2\cos\theta}{\cancel{r^2}}\right)\right] + \frac{1}{r\,\text{sen}\,\theta}\frac{\partial}{\partial \theta}\left[\frac{\text{sen}\,\theta}{r^3} \cdot \text{sen}\,\theta\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{2\cos\theta}{r}\right) + \frac{1}{r\,\text{sen}\,\theta} \cdot \frac{1}{r^3}\frac{\partial}{\partial \theta}(\text{sen}^2\theta)$$

$$= \frac{1}{r^2}(2\cos\theta)\left(-\frac{1}{r^2}\right) + \frac{1}{r^4\,\text{sen}\,\theta}\frac{\partial}{\partial \theta}\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$$

$$= -\frac{2}{r^4}\cos\theta + \frac{1}{r^4\,\text{sen}\,\theta}\left[-\frac{1}{\cancel{2}}(-\cancel{2}\,\text{sen}\,2\theta)\right]$$

$$= -\frac{2}{r^4}\cos\theta + \frac{1}{r^4\,\text{sen}\,\theta}(\text{sen}\,2\theta)$$

$$= -\frac{2}{r^4}\cos\theta + \frac{1}{r^4\,\cancel{\text{sen}\,\theta}}[2\,\cancel{\text{sen}\,\theta}\cos\theta]$$

$$= -\frac{2}{r^4}\cos\theta + \frac{2}{r^4}\cos\theta$$

$$= 0$$