① $\vec{A} = 2\vec{a_x} + 5\vec{a_y} - 3\vec{a_z}$

$\vec{B} = 3\vec{a_x} - 4\vec{a_y}$

$\vec{C} = 5\vec{a_x} - 2\vec{a_y} + 7\vec{a_z}$

a) $\vec{A} + 2B = (2, 5, -3) + 2(3, -4, 0)$

$= (2, 5, -3) + (6, -8, 0)$

$= 8, -3, -3$

b) $|\vec{A} - 5\vec{C}| = |(2, 5, -3) - 5(5, -2, 7)|$

$= |(2, 5, -3) - (25, -10, 35)|$

$= |(-23, 15, -38)|$

$= \sqrt{(-23)^2 + (15)^2 + (-38)^2}$

$= 46.88$

c) $|k\vec{B}| = 2 = |(3K, -4K, 0)|$

$= \sqrt{(3K)^2 + (-4K)^2}$

$= \sqrt{9K^2 + 16K^2}$

$= \sqrt{25K^2}$

$= 5K = 2$

$K = \frac{2}{5}$

$K = 0.4$

d) $\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 2 & 5 & -3 \\ 3 & -4 & 0 \end{vmatrix}$

$$= -12\,\vec{a_x} - 9\,\vec{a_y} + (-8-15)\,\vec{a_z}$$

$$= -12\vec{a_x} - 9\vec{a_y} - 23\vec{a_z}$$

$$\vec{A} \cdot \vec{B} = (2)(3) + (5)(-4) + (-3)(0)$$

$$= 6 - 20$$

$$= -14$$

$$\frac{(\vec{A} \times \vec{B})}{\vec{A} \cdot \vec{B}} = \frac{-12\vec{a_x} - 9\vec{a_y} - 23\vec{a_z}}{-14}$$

$$= 0.857\,\vec{a_x} + 0.643\vec{a_y} + 1.643\,\vec{a_z}$$

② $\vec{A} = \alpha \vec{a_x} + \vec{a_y} + 4\vec{a_z}$

$\vec{B} = 3\vec{a_x} + \beta\vec{a_y} + 6\vec{a_z}$

$\vec{C} = 5\vec{a_x} - 2\vec{a_y} + \gamma\vec{a_z}$

PARA QUE SEAN MUTUAMENTE ORTOGONALES:

$\vec{A}\cdot\vec{B} = 0 = 3\alpha + \beta + 24$

$\vec{A}\cdot\vec{C} = 0 = 5\alpha - 2 + 4\gamma$

$\vec{B}\cdot\vec{C} = 0 = 15 - 2\beta + 6\gamma$

$$\begin{cases} 3\alpha + \beta = -24 \\ 5\alpha + 4\gamma = 2 \\ -2\beta + 6\gamma = -15 \end{cases}$$

$$\alpha = \frac{\begin{vmatrix} -24 & 1 & 0 \\ 2 & 0 & 4 \\ -15 & -2 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & 0 \\ 5 & 0 & 4 \\ 0 & -2 & 6 \end{vmatrix}} = \frac{-24(8) - 1(12+60)}{3(8) - 1(30)} = \frac{-264}{-6} = 44$$

$$\beta = \frac{\begin{vmatrix} 3 & -24 & 0 \\ 5 & 2 & 4 \\ 0 & -15 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & 0 \\ 5 & 0 & 4 \\ 0 & -2 & 6 \end{vmatrix}} = \frac{3(12+60) - 5(-24*6)}{3(8) - 1(30)} = \frac{936}{-6} = -156$$

$$\gamma = \frac{\begin{vmatrix} 3 & 1 & -24 \\ 5 & 0 & 2 \\ 0 & -2 & -15 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & 0 \\ 5 & 0 & 4 \\ 0 & -2 & 6 \end{vmatrix}} = \frac{3(4) - 5(-15 - 48)}{-6} = \frac{327}{-6} = -54.5$$

③ $\vec{D} = (x+z)\,\vec{a}_y$

$A_x = 0$

$A_y = x + z$

$A_z = 0$

ESFÉRICAS:

$$\therefore D_r = A_x \cancelto{0}{\operatorname{sen}\theta\cos\phi} + A_y \operatorname{sen}\theta \operatorname{sen}\phi + A_z \cancelto{0}{\cos\theta}$$

$$= (x+z)\operatorname{sen}\theta\operatorname{sen}\phi$$

$$= (r\operatorname{sen}\theta\cos\phi + r\cos\theta)\operatorname{sen}\theta\operatorname{sen}\phi$$

$$D_\theta = A_x \cancelto{0}{\cos\theta\cos\phi} + A_y \cos\theta\operatorname{sen}\phi - A_z \cancelto{0}{\operatorname{sen}\theta}$$

$$= (x+z)\cos\theta\operatorname{sen}\phi$$

$$= (r\operatorname{sen}\theta\cos\phi + r\cos\theta)\cos\theta\operatorname{sen}\phi$$

$$D_\phi = -A_x \cancelto{0}{\operatorname{sen}\phi} + A_y\cos\phi$$

$$= (x+z)\cos\phi$$

$$= (r\operatorname{sen}\theta\cos\phi + r\cos\theta)\cos\phi$$

CILINDRICAS:

$$D_\rho = A_x \cancelto{0}{\cos\phi} + A_y\operatorname{sen}\phi$$

$$= (x+z)\operatorname{sen}\phi$$

$$= (\rho\cos\phi + z)\operatorname{sen}\phi$$

$$D_\phi = -A_x \cancelto{0}{\operatorname{sen}\phi} + A_y\cos\phi$$

$$= (x+z)\cos\phi$$

$$= (\rho\cos\phi + z)\cos\phi$$

$$D_z = A_z = 0$$

$$\vec{E} = \underbrace{(y^2 - x^2)}_{A_x}\vec{a_x} + \underbrace{xyz}_{A_y}\,\vec{a_y} + \underbrace{(x^2 - z^2)}_{A_z}\,\vec{a_z}$$

CILÍNDRICAS:

$$E_\rho = A_x \cos\phi + A_y \operatorname{sen}\phi$$
$$= (y^2 - x^2)\cos\phi + (xyz)\operatorname{sen}\phi$$
$$= \rho^2(\operatorname{sen}^2\phi - \cos^2\phi)\cos\phi + \rho^2 z \operatorname{sen}^2\phi \cos\phi$$
$$= -\rho^2 \cos 2\phi \cos\phi + \rho^2 z \operatorname{sen}^2\phi \cos\phi$$

$$E_\phi = -A_x \operatorname{sen}\phi + A_y \cos\phi$$
$$= -(y^2 - x^2)\operatorname{sen}\phi + (xyz)\cos\phi$$
$$= (x^2 - y^2)\operatorname{sen}\phi + (xyz)\cos\phi$$
$$= \rho^2(\cos^2\phi - \operatorname{sen}^2\phi)\operatorname{sen}\phi + \rho^2 z \cos^2\phi \operatorname{sen}\phi$$
$$= \rho^2 \cos 2\phi \operatorname{sen}\phi + \rho^2 z \cos^2\phi \operatorname{sen}\phi$$

$$E_z = (x^2 - z^2)$$
$$= \rho^2 \cos^2\phi - z^2$$

ESFÉRICAS:

$$E_r = A_x \operatorname{sen}\theta\cos\phi + A_y \operatorname{sen}\theta\operatorname{sen}\phi + A_z\cos\theta$$
$$= (y^2 - x^2)\operatorname{sen}\theta\cos\phi + xyz\operatorname{sen}\theta\operatorname{sen}\phi + (x^2 - z^2)\cos\theta$$
$$= r^2[\operatorname{sen}^2\theta(\operatorname{sen}^2\phi - \cos^2\phi)]\operatorname{sen}\theta\cos\phi$$
$$+ r^3\operatorname{sen}^3\theta\operatorname{sen}^2\phi\cos\theta\cos\phi$$
$$+ (r^2\operatorname{sen}^2\theta\cos^2\phi - r^2\cos^2\theta)\cos\theta$$

$$E_\theta = A_x \cos\theta\cos\phi + A_y \cos\theta \operatorname{sen}\phi - A_z \operatorname{sen}\theta$$

$$= (y^2 - x^2)\cos\theta\cos\phi + xyz\cos\theta\operatorname{sen}\phi - (x^2 - z^2)\operatorname{sen}\theta$$

$$= r^2[\operatorname{sen}^2\theta(\operatorname{sen}^2\phi - \cos^2\phi)]\cos\theta\cos\phi$$

$$+ r^3 \operatorname{sen}^2\theta\cos^2\theta\operatorname{sen}^2\phi\cos\phi$$

$$- r^2(\operatorname{sen}^2\theta\cos^2\phi - \cos^2\theta)\operatorname{sen}\theta$$

$$E_\phi = -A_x \operatorname{sen}\phi + A_y \cos\phi$$

$$= -(y^2 - x^2)\operatorname{sen}\phi + xyz\cos\phi$$

$$= -r^2[\operatorname{sen}^2\theta(\operatorname{sen}^2\phi - \cos^2\phi)]\operatorname{sen}\phi$$

$$+ r^3\operatorname{sen}^2\theta\cos\theta\cos^2\phi\operatorname{sen}\phi$$

④ $\vec{H} = -5\rho \operatorname{sen}\phi\, \vec{a}_\rho - \rho z \cos\phi\, \vec{a}_\phi + 2\rho\, \vec{a}_z$

EN $P(2, 30°, -1)$

$$= -5(2)\operatorname{sen}(30°)\,\vec{a}_\rho - (2)(-1)\cos(30°)\,\vec{a}_\phi + 2(2)\,\vec{a}_z$$

$$= -5(2)\left(\tfrac{1}{2}\right)\vec{a}_\rho - 2(-1)\left(\tfrac{\sqrt{3}}{2}\right)\vec{a}_\phi + 2(2)\,\vec{a}_z$$

$$= -5\,\vec{a}_\rho + \sqrt{3}\,\vec{a}_\phi + 4\,\vec{a}_z$$

a) $$|\vec{H}| = \sqrt{(-5)^2 + (\sqrt{3})^2 + (4)^2}$$

$$= \sqrt{25 + 3 + 16}$$

$$= \sqrt{44}$$

$$\vec{a} = \frac{-5}{\sqrt{44}}\,\vec{a}_\rho + \frac{\sqrt{3}}{\sqrt{44}}\,\vec{a}_\phi + \frac{4}{\sqrt{44}}\,\vec{a}_z$$

$$= -0.7538\,\vec{a}_\rho + 0.2611\,\vec{a}_\phi + 0.6030\,\vec{a}_z$$

b) $$\vec{H}_x = (H_\rho \cos\phi - H_\phi \operatorname{sen}\phi)\,\vec{a}_x$$

$$= (-5\rho \operatorname{sen}\phi \cos\phi + \rho z \cos\phi \operatorname{sen}\phi)\,\vec{a}_x$$

$$= [-5(2)\operatorname{sen}(30°)\cos(30°) + (2)(-1)\cos(30°)\operatorname{sen}(30°)]\,\vec{a}_x$$

$$= \left[-5(\cancel{2})\left(\tfrac{1}{\cancel{2}}\right)\left(\tfrac{\sqrt{3}}{2}\right) + (\cancel{2})(-1)\left(\tfrac{\sqrt{3}}{\cancel{2}}\right)\left(\tfrac{1}{2}\right)\right]\vec{a}_x$$

$$= \left(-\frac{5\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)\vec{a}_x$$

$$= \left(-\frac{6\sqrt{3}}{2}\right)\vec{a}_x$$

$$= (-3\sqrt{3})\,\vec{a}_x$$

$$= (-5.196)\,\vec{a}_x$$

COMPONENTE PARALELA $\vec{H}_{x_p} = P\,\vec{H}_x$. SI $P = 2$

$$\vec{H}_{x_p} = 2(-5.196)\,\vec{a}_x$$

$$= -10.3923$$

c) $\vec{H}_n = \vec{H}_\rho \, \vec{a}_\rho$

POR EJEMPLO:

$\vec{H}_n = -0.7538 \, \vec{a}_\rho$

d) TANGENCIAL A $\phi = 30°$

$\vec{H}_t = H_\rho \, \vec{a}_\rho + H_z \, \vec{a}_z$

$= -0.7538 \, \vec{a}_\rho + 0.6030 \, \vec{a}_z$