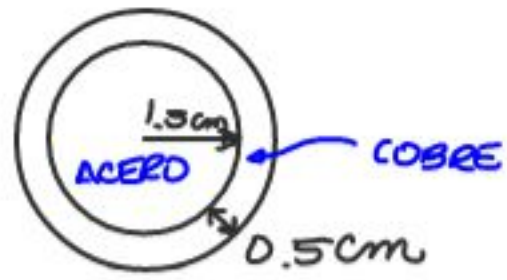
① $L = 10m$

$R_{acero} = 1.5\,cm$



$Grosor_{COBRE} = 0.5cm$

$\rho_{COBRE} = 1.77X10^{-8}\,\Omega m$

$\rho_{ACERO} = 11.8X10^{-8}\,\Omega m$

a) $R_{COND} = ?$

$$S_{ACERO} = \pi R_{acero}^2$$
$$= \pi (1.5X10^{-2})^2$$
$$= 7.07X10^{-4}\,m^2$$

$$S_{COBRE} = \pi R_{COBRE}^2$$
$$= \pi (2X10^{-2})^2 - \pi (1.5X10^{-2})^2$$
$$= 5.50X10^{-4}\,m^2$$

$$R = \frac{\rho_c L}{S}$$

$$R_{COND} = \frac{R_{COBRE} R_{ACERO}}{R_{COBRE} + R_{ACERO}} = \frac{\left(\frac{\rho_{c_{COBRE}} L}{S_{COBRE}}\right)\left(\frac{\rho_{c_{ACERO}} L}{S_{ACERO}}\right)}{\left(\frac{\rho_{c_{COBRE}}}{S_{COBRE}} + \frac{\rho_{c_{ACERO}}}{S_{ACERO}}\right) L}$$

$$= \frac{\rho_{c_{COBRE}}\,\rho_{c_{ACERO}}}{\rho_{c_{COBRE}} S_{ACERO} + \rho_{c_{ACERO}} S_{COBRE}} L$$

$$= \frac{(1.77X10^{-8})(11.8X10^{-8})(10)}{(1.77X10^{-8})(7.07X10^{-4}) + (11.8X10^{-8})(5.50X10^{-4})}$$

$$= 2.70X10^{-4}\,\Omega$$

b) $I_{COND} = 60A$ $\quad V = I_{COBRE} R_{COBRE} = I_{ACERO} R_{ACERO}$

$$\frac{I_{COBRE}}{I_{ACERO}} = \frac{R_{ACERO}}{R_{COBRE}} = \frac{\rho_{C_{ACERO}} / S_{ACERO}}{\rho_{C_{COBRE}} / S_{COBRE}}$$

$$= \frac{11.8\times10^{-8} / 7.07\times10^{-4}}{1.77\times10^{-8} / 5.50\times10^{-4}}$$

$$= 5.19$$

$I_{COBRE} + I_{ACERO} = 60A$

$5.19 I_{ACERO} + I_{ACERO} = 60A$

$$I_{ACERO} = \frac{60}{6.19}$$

$I_{ACERO} = 9.69\ A$

$I_{COBRE} = 5.19 I_{ACERO}$

$= 5.19(9.69A)$

$= 50.31\ A$

c) $R = \frac{\rho L}{S} = \frac{(1.77\times10^{-8})(10)}{5.5\times10^{-4}} = 3.22\times10^{-4}\ \Omega$

② $\mathcal{E} = 2.4\,\mathcal{E}_0$

$V = 300z^2$

a) $\mathbf{D} = \mathcal{E}_0 \mathcal{E}_r \mathbf{E}$

$= (8.854\times10^{-12})(2.4)(-600z)\,a_z$

$= -1.27\times10^{-8}\, z\, a_z \; nC/m^2$

$\mathbf{E} = -\nabla V = -\frac{dV}{dz}\, a_z$

$= -\frac{d}{dz}(300z^2)\, a_z$

$= -600z\, a_z$

$\rho_v = \nabla \cdot D$

$= \frac{\partial}{\partial z}(1.27\times10^{-8} z)$

$= -1.27\times10^{-8} \; C/m^3$

b) $\mathbf{P} = \chi_e \mathcal{E}_0 \mathbf{E}$ $\qquad \mathcal{E}_r = 1 + \chi_e \Rightarrow \chi_e = \mathcal{E}_r - 1$

$= (2.4 - 1)(8.854\times10^{-12})(-600z)\, a_z$

$= -7.43\, a_z \; nC/m^2$

$\rho_{PV} = \nabla \cdot \mathcal{E}_0 \mathbf{E} - \nabla \cdot \mathbf{D}$

$= -600(8.854\times10^{-12}) + 1.27\times10^{-8}$

$= 7.39\times10^{-9} \; nC/m^3$

③ $J = \frac{100}{\rho^2} a_z \ A/m^2$

a) $\nabla . J = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{100}{\rho^2} \right) = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\frac{100}{\rho} \right) = \frac{1}{\rho} \left(- \frac{100}{\rho^2} \right)$

$= - \frac{100}{\rho^3}$

$- \frac{\partial \rho_v}{\partial t} = \nabla . J = - \frac{100}{\rho} \ C/m^3 . s$

$\frac{\partial \rho_v}{\partial t} = \frac{100}{\rho^3} \ C/m^3 . s$

b) $I = \int J . ds$

$= \int_{\phi=0}^{2\pi} \int_{z=0}^{1} \rho \frac{100}{\rho^2} \, dz \, d\phi \quad ; \rho = 2$

$= \frac{100}{\rho} z \Big|_{z=0}^{1} \phi \Big|_{\phi=0}^{2\pi}$

$= \frac{100}{\rho} (1)(2\pi)$

$= \frac{100}{2} (1)(2\pi)$

$= 100\pi$

$= 314.16 \ A$

④ R_1 ($\rho \leq 4cm$) $\varepsilon_{r_1} = 3.5$

R_2 ($\rho \geq 4cm$) $\varepsilon_{r_2} = 1.5$

$\mathbf{D}_2 = 12\,a_\rho - 6\,a_\phi + 9\,a_z \ nC/m^2$

a) $\mathbf{E}_1 = ?$

$\mathbf{D}_1 = ?$

$D_{2n} = 12\,a_\rho = D_{1n}$ $\qquad D_{2t} = -6\,a_\phi + 9\,a_z$

$E_{1t} = E_{2t}$

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

$$D_{1t} = \frac{\varepsilon_1}{\varepsilon_2} D_{2t}$$

$$= \frac{3.5}{1.5}(-6\,a_\phi + 9\,a_z)$$

$$= (-14\,a_\phi + 21\,a_z)\ nC/m^2$$

$$\mathbf{D}_1 = D_{1t} + D_{1n}$$

$$= (12\,a_\rho - 14\,a_\phi + 21\,a_z)\ nC/m^2$$

$$\mathbf{E}_1 = \frac{D_1}{\varepsilon_1}$$

$$= \frac{(12\,a_\rho - 14\,a_\phi + 21\,a_z)\,nC/m^2}{3.5\,(8.854 \times 10^{-12})}$$

$$= (387.23\,a_\rho - 451.77\,a_\phi + 677.66\,a_z)\ V/m$$

b) $P_2 = ?$

$\rho_{pv_2} = ?$

$P_2 = \varepsilon_0 \chi_{e_2} E_2$

$= \varepsilon_0 (1.5-1) \frac{D_2}{\varepsilon_2}$

$= (8.854 \times 10^{-12})(1.5-1)(12a_\rho - 6a_\phi + 9a_z)/(1.5\varepsilon_0)$

$= (4a_\rho - 2a_\phi + 3a_z)\ nC/m^2$

$\rho_{pv_2} = \nabla \cdot P_2 = \frac{1}{\rho}\frac{\partial}{\partial \rho}(4\rho) = \frac{1}{\rho}(4) = \frac{4}{\rho}$

c) $W_{E_1} = \frac{1}{2} D_1 \cdot E_1$

$= \frac{1}{2}[(12\times10^{-9})(387.23) + (14\times10^{-9})(451.77) + (21\times10^{-9})(677.66)]$

$= 126.01\ mJ/m^2$

$W_{E_2} = \frac{1}{2} D_2 \cdot \frac{D_2}{\varepsilon_2}$

$= \frac{1}{2}[(12\times10^{-9})(12\times10^{-9}) + (6\times10^{-9})(6\times10^{-9}) + (9\times10^{-9})(9\times10^{-9})]/(1.5\varepsilon_0)$

$= 9.83\ \mu J/m^2$