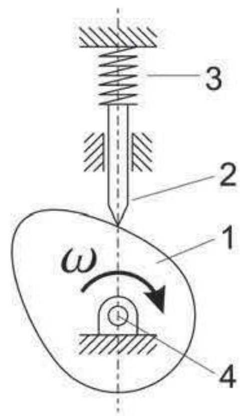

FIGURE 4.1 Cam with a knife-edge follower.

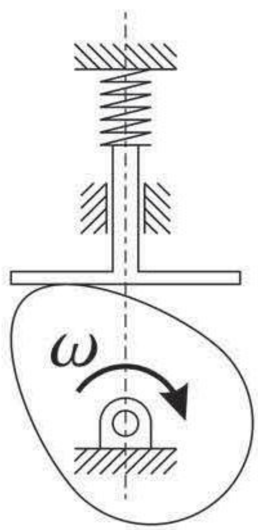

FIGURE 4.2 Cam with a flat-faced follower.

4.2 CIRCULAR CAM PROFILE

A *circular cam* is made by mounting a circular plate on a camshaft at some distance d away from the circle center (Figure 4.8). This gives the simplest cam profile. The eccentric attachment of the circular plate produces a reciprocating motion of the follower. The problem of direct analysis is to find the follower displacement given the rotation of a cam with known radius R and eccentricity d.

Place the origin of the global coordinate system in the camshaft center. Then, the distance D from this center (Figure 4.9) characterizes the follower position. The tip of the follower can also be reached following the vectors **d** and **R**. The three

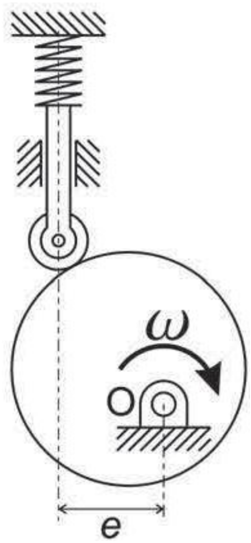

FIGURE 4.3 Cam with a roller follower.

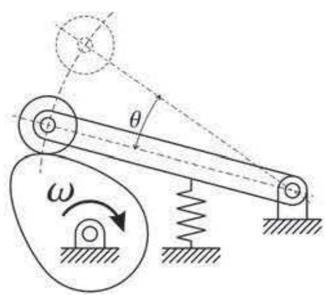

FIGURE 4.4 Cam with a rocking follower.

vectors, **d**, **R**, and **D**, form a loop at any cam position. This loop is identical to a loop for a slider-crank mechanism, in which d plays the role of a crank, R of a connecting rod, and D characterizes the slider position. This analogy means that the analysis of this cam mechanism is identical to that of the slider-crank mechanism. Indeed, the loop-closure equation in this case is

$$d[\cos\gamma, \sin\gamma]^T + R[\cos\theta, \sin\theta]^T + D\left[\cos\left(\frac{3\pi}{2}\right), \sin\left(\frac{3\pi}{2}\right)\right]^T = 0 \qquad (4.1)$$

where the angles θ and γ are shown in Figure 4.9. In the above equation the unknowns are the distance D and the angle γ. Thus, this equation falls into the second case category according to the analysis of various cases in Chapter 2. From the equivalency of Equations 2.28 and 4.1, it follows that the corresponding solutions for the former, namely, Equations 2.31 and 2.32, can be used in this case. It is necessary

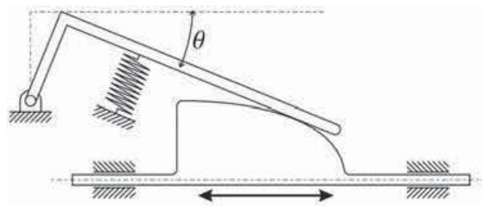

FIGURE 4.5 Reciprocating cam with a flat follower.

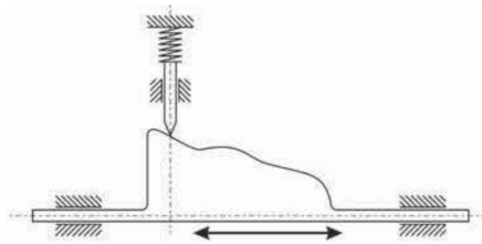

FIGURE 4.6 Reciprocating cam with a knife follower.

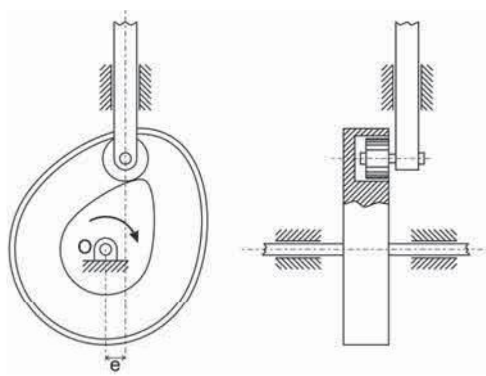

FIGURE 4.7 Cam with a roller inside a groove.

to substitute in these solutions r_i by D, θ_j by θ, θ_i by $3\pi/2$, α by γ, r_j by R, and b by $-d$. As a result, the solution for the follower displacement is

$$D = d\sin\gamma \pm \sqrt{R^2 - d^2\cos^2\gamma} \tag{4.2}$$

and the angle θ is

$$\theta = \begin{cases} \theta^* & \text{if } -\sin\theta > 0 \text{ and } \cos\theta < 0 \\ \pi - \theta^* & \text{if } -\sin\theta < 0 \text{ and } \cos\theta < 0 \\ \pi + \theta^* & \text{if } -\sin\theta < 0 \text{ and } \cos\theta > 0 \\ 2\pi - \theta^* & \text{if } -\sin\theta > 0 \text{ and } \cos\theta > 0 \end{cases} \tag{4.3}$$

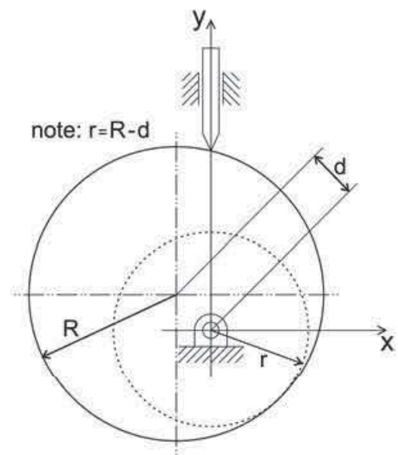

FIGURE 4.8 Circular cam.

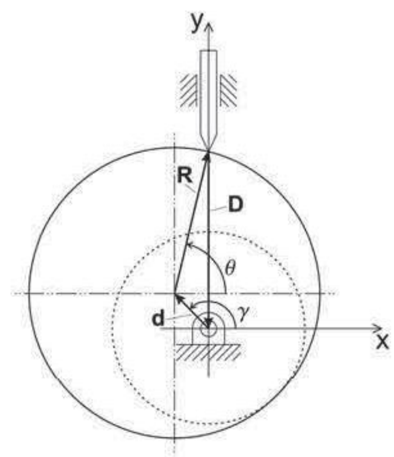

FIGURE 4.9 Loop-closure equation for a circular cam.

where

$$\theta^* = \frac{3\pi}{2} + \arcsin\left|\frac{d\cos\gamma}{R}\right| \tag{4.4}$$

$$\cos\theta = -\frac{d}{R}\cos\gamma \tag{4.5}$$

and

$$\sin\theta = -\frac{d}{R}\sin\gamma + \frac{D}{R} \tag{4.6}$$

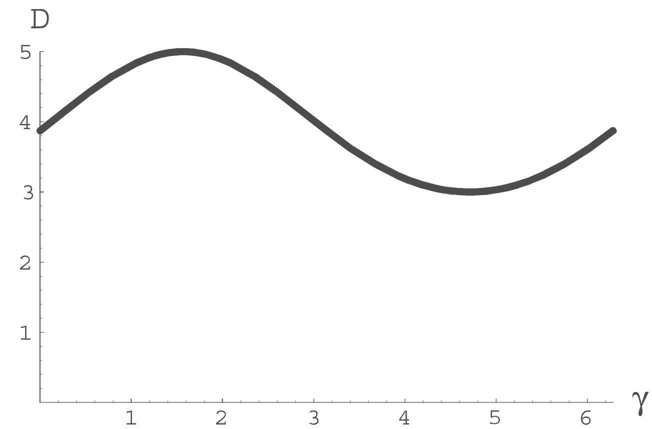

FIGURE 4.10 Position of the follower during one cycle of circular cam rotation.

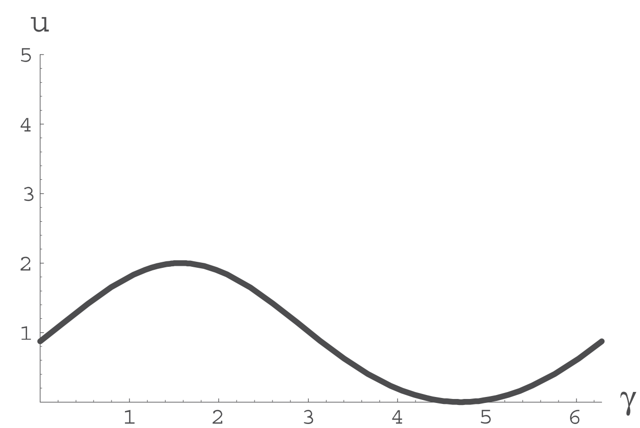

FIGURE 4.11 Displacement diagram for a circular cam.

If, at the extreme, $d = R$, then Equation 4.2 gives $D = R\sin\theta \pm R\sin\theta$. It follows from the above that the correct sign in Equation 4.2 must be plus for this case. At the other extreme, if $d << R$, then the second term under the square root can be neglected and the result is $D = d\sin\gamma \pm R$. Again, the correct sign must be plus for D to be positive. Thus, one can assume that the sign in Equation 4.2 must be plus for any value of d.

$$D = d\sin\gamma + \sqrt{R^2 - d^2\cos^2\gamma} \tag{4.7}$$

In Figure 4.10 the position of the follower, D, is shown for one cycle of cam rotation for the case of d = 1 cm and R = 4 cm. The difference between the maximum follower displacement and its minimum is called the *lift.* If the minimum follower coordinate (position) is subtracted from its current position, the resulting diagram

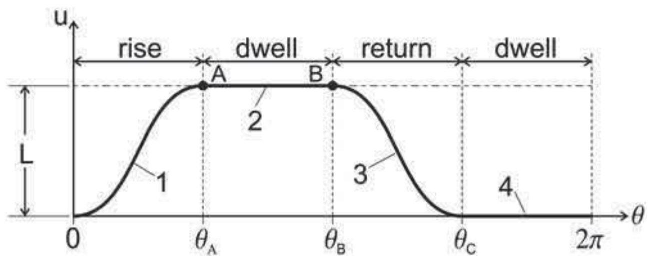

FIGURE 4.12 Typical displacement diagram.

is called the *displacement diagram*. Such a diagram for a circular cam is shown in Figure 4.11.

A minimum follower position constitutes a reference level for the follower displacement. This reference level is a circle with a constant radius $r_b = D - D_{min}$, which is called the *base radius*. Thus, a cam profile can be viewed as a displacement diagram wrapped around the base circle.

4.3 DISPLACEMENT DIAGRAM

The displacement diagram serves as an input into the cam mechanism design. Consider the cam mechanism in Figure 1.1. The function of the cam might be, as is the case in the internal combustion engine, to open the valve, to keep it open during some part of the cycle (this is called *dwell*), and then to close it and to keep it closed for some time (to dwell again). A generic displacement diagram may look as shown in Figure 4.12. The requirements of how long it should take to rise, to dwell, to return, and to dwell again, and also of what the lift should be define the size and the shape of the cam. The function depicted in Figure 4.12 is a piecewise function, which means that special attention should be paid to transition from one continuous function to another, for example, from rise to dwell. This represents another objective of cam design, *to ensure a smooth transition of the follower from one part of the displacement diagram to another.*

Consider, for example, a transition from rise to dwell in Figure 4.12. The rise curve is described by some function $u_1(\theta)$, while the dwell is described by another function $u_2(\theta) = \text{const}$. For a smooth transition from the rise to dwell, it is needed that at $\theta = \theta_A$,

$$u_1(\theta_A) = u_2(\theta_A), \frac{du_1(\theta)}{d\theta} = 0, \text{and } \frac{d^2u_1(\theta)}{d\theta^2} = 0 \tag{4.8}$$

Since $\theta = \omega t$ (where ω is the angular velocity of the cam, and t is the time), the requirements for the equality of first and second derivatives is equivalent to the requirements that the velocity and acceleration of the follower does not experience jumps at the point of transition. The same requirement should be met at the other transitional points in Figure 4.12: $\theta = 0$, $\theta = \theta_B$, and $\theta = \theta_C$. These latter requirements put limitations on what type of functions can be used to generate the