

FIGURE 2.40 Components of acceleration vector.

One can rewrite the above equation in such a way that the directions of all vectors are referenced with respect to the direction of vector \mathbf{r} . Then Equation 2.112 takes the form

$$\begin{aligned} \ddot{\mathbf{r}} = & \dot{r}[\cos \theta, \sin \theta]^T + 2\dot{r}\left[\cos\left(\theta + \frac{\pi}{2}\right), \sin\left(\theta + \frac{\pi}{2}\right)\right]^T \omega \\ & - r[\cos(\theta + \pi), \sin(\theta + \pi)]^T \omega^2 + r\left[\cos\left(\theta + \frac{\pi}{2}\right), \sin\left(\theta + \frac{\pi}{2}\right)\right]^T \alpha \end{aligned} \quad (2.113)$$

The directions of four vectors and their magnitudes are shown in Figure 2.40, where one can see that the direction of first vector coincides with the direction of vector \mathbf{r} ; the direction of third vector is opposite to that of vector \mathbf{r} , and the two other vectors are perpendicular to vector \mathbf{r} whereas their directions are found by rotating vector \mathbf{r} counterclockwise by $\pi/2$. The first vector in Equation 2.113 is called the *translational* component of acceleration, the third is called the *centripetal* component of acceleration, the fourth is called the *angular* component of acceleration, and the second is called the *coriolis* component of acceleration.

Note that centripetal acceleration is caused by the rotation of the vector (irrespective of whether this rotation is time dependent or time independent), whereas coriolis acceleration is caused by the rotation of a translationary moving vector. Both of these components are functions of velocities only, and thus can be found based on the velocity analysis. The other two components of acceleration, translational and angular, are found as a result of acceleration analysis.

2.5.2 EQUATIONS FOR ACCELERATIONS

The equations for accelerations follow from the loop-closure equation for positions, Equation 2.15, if the equation is differentiated twice with respect to time. As a result, the loop-closure equation for accelerations is obtained. Note that it is assumed that all parameters are time-dependent variables.

$$\begin{aligned} \sum_{i=1}^N \ddot{r}_i[\cos \theta_i, \sin \theta_i]^T + 2\dot{r}_i[-\sin \theta_i, \cos \theta_i]^T \omega_i \\ - r_i[\cos \theta_i, \sin \theta_i]^T \omega_i^2 + r_i[-\sin \theta_i, \cos \theta_i]^T \alpha_i = 0 \end{aligned} \quad (2.114)$$

In Equation 2.114 the unknowns are \ddot{r}_i and α_i , and the system defining them is linear. As before, the loop-closure equation for accelerations can have only two unknowns. This again entails five possible combinations of these unknowns. In this case the solutions for each can be found from the solutions for velocities in a straightforward manner.

First Case

The unknowns, $\ddot{\theta}_j$ and \ddot{r}_j , are found by differentiating Equations 2.77 and 2.78 with respect to time assuming that all the variables are time dependent. The results, taking into account Equations 2.77 and 2.78, are

$$\alpha_j = \frac{1}{r_j} [-\ddot{b}_x \sin \theta_j + \ddot{b}_y \cos \theta_j - 2\dot{r}_j \omega_j] \quad (2.115)$$

and

$$\ddot{r}_j = \ddot{b}_x \cos \theta_j + \ddot{b}_y \sin \theta_j + r_j \omega_j^2 \quad (2.116)$$

Second Case

In this case the two unknowns, $\ddot{\theta}_j$ and \ddot{r}_i , are found by differentiating Equations 2.80 and 2.81, respectively.

$$\alpha_j = \frac{1}{r_j \cos(\theta_j - \theta_i)} [((\omega_j - \omega_i)r_j \omega_j - \dot{r}_j) \sin(\theta_j - \theta_i) - (2\omega_j - \omega_i)\dot{r}_j \cos(\theta_j - \theta_i) - (\omega_i \dot{b}_y + \ddot{b}_x) \sin \theta_i + (-\omega_i \dot{b}_x + \ddot{b}_y) \cos \theta_i - \omega_i \dot{r}_i - r_i \alpha_i] \quad (2.117)$$

and

$$\ddot{r}_i = \frac{1}{\cos(\theta_j - \theta_i)} [(\dot{r}_i(\omega_j - 2\omega_i) - r_i \alpha_i) \sin(\theta_j - \theta_i) - r_i \omega_i(\omega_j - \omega_i) \cos(\theta_j - \theta_i) - \dot{r}_j + (\ddot{b}_x + \dot{b}_y \omega_j) \cos \theta_j + (\ddot{b}_y - \dot{b}_x \omega_j) \sin \theta_j] \quad (2.118)$$

Third Case

The unknowns in this case are the translational accelerations, \ddot{r}_i and \ddot{r}_j . The expressions for them are found by differentiating Equations 2.82 and 2.83.

$$\ddot{r}_i = \frac{1}{\sin(\theta_i - \theta_j)} [-(\dot{r}_i(2\omega_i - \omega_j) + r_i \alpha_i) \cos(\theta_i - \theta_j) + r_i \omega_i(\omega_i - \omega_j) \sin(\theta_i - \theta_j) - \dot{r}_j \omega_j - r_j \alpha_j - (\ddot{b}_x + \dot{b}_y \omega_j) \sin \theta_j + (\ddot{b}_y - \dot{b}_x \omega_j) \cos \theta_j] \quad (2.119)$$

and the equation for \ddot{r}_j is obtained by interchanging indices i and j in the above.

$$\ddot{r}_j = \frac{1}{\sin(\theta_j - \theta_i)} [-(\dot{r}_j(2\omega_j - \omega_i) + r_j \alpha_j) \cos(\theta_j - \theta_i) + r_j \omega_j(\omega_j - \omega_i) \sin(\theta_j - \theta_i) - \dot{r}_i \omega_i - r_i \alpha_i - (\ddot{b}_x + \dot{b}_y \omega_i) \sin \theta_i + (\ddot{b}_y - \dot{b}_x \omega_i) \cos \theta_i] \quad (2.120)$$

Fourth Case

The angular accelerations are found by differentiating Equations 2.84 and 2.85.

$$\alpha_i = \frac{1}{r_i \sin(\theta_j - \theta_i)} [(-\omega_i r_i (\omega_j - \omega_i) - \dot{r}_i) \cos(\theta_j - \theta_i) + \dot{r}_i (\omega_j - 2\omega_i) \sin(\theta_j - \theta_i) - \ddot{r}_j + (\ddot{b}_x + \dot{b}_y \omega_j) \cos \theta_j + (\dot{b}_y - \dot{b}_x \omega_j) \sin \theta_j] \quad (2.121)$$

and the equation for α_j is obtained by interchanging indices i and j in the above.

$$\alpha_j = \frac{1}{r_j \sin(\theta_i - \theta_j)} [(-\omega_j r_j (\omega_i - \omega_j) - \dot{r}_j) \cos(\theta_i - \theta_j) + \dot{r}_j (\omega_i - 2\omega_j) \sin(\theta_i - \theta_j) - \ddot{r}_i + (\ddot{b}_x + \dot{b}_y \omega_i) \cos \theta_i + (\dot{b}_y - \dot{b}_x \omega_i) \sin \theta_i] \quad (2.122)$$

Fifth Case

In this case the accelerations, $\ddot{\theta}_i$ and \ddot{r}_j , are found by differentiating Equations 2.90 and 2.91 taking into account that γ and β are constants.

$$\alpha_i = \frac{A_i \cos(\theta_i - \gamma) - B_i \sin(\theta_i - \gamma) - \dot{r}_i \sin \gamma - \dot{r}_k \sin(\gamma - \beta)}{-d_x \sin(\theta_i - \gamma) + d_y \cos(\theta_i - \gamma)} \quad (2.123)$$

and

$$\ddot{r}_j = \frac{1}{T_j} [-A_j \cos(\theta_i - \gamma) - B_j \sin(\theta_i - \gamma) + C_j \cos \theta_i + D_j \sin \theta_i + K_j \cos(\theta_i - \beta) + L_j \sin(\theta_i - \beta) + Q_j] \quad (2.124)$$

where it is denoted

$$A_i = -\dot{d}_y \omega_i + d_x \omega_i^2 - \dot{b}_x \omega_i + \ddot{b}_y \quad (2.125)$$

$$B_i = -\dot{d}_x \omega_i + d_y \omega_i^2 - \dot{b}_y \omega_i + \ddot{b}_x \quad (2.126)$$

$$A_j = (-\dot{d}_y + d_x \omega_i) \dot{r}_j \quad (2.127)$$

$$B_j = (\dot{d}_x + d_y \omega_i) \dot{r}_j \quad (2.128)$$

$$C_j = \dot{r}_i d_y + \dot{r}_i \dot{d}_i - \dot{r}_i d_x \omega_i \quad (2.129)$$

$$D_j = -\dot{r}_i d_x - \dot{r}_i \dot{d}_x - \dot{r}_i d_y \omega_i \quad (2.130)$$

$$K_j = \dot{r}_k d_y + \dot{r}_k \dot{d}_y - \dot{r}_k d_x \omega_i \quad (2.131)$$

$$L_j = -\dot{r}_k d_x - \dot{r}_k \dot{d}_x - \dot{r}_k d_y \omega_i \quad (2.132)$$

$$Q_j = -\dot{d}_y \dot{b}_x - d_y \ddot{b}_x + \dot{d}_x \dot{b}_y + d_x \ddot{b}_y \quad (2.133)$$

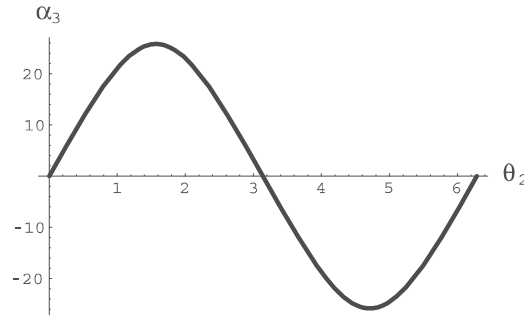


FIGURE 2.41 Angular acceleration of the connecting rod vs. crank angle.

$$T_j = -d_y \cos(\theta_i - \gamma) + d_x \sin(\theta_i - \gamma) \quad (2.134)$$

where b_x , b_y are given by Equation 2.45, and d_x , d_y are defined by Equations 2.88 and 2.89, respectively.

2.5.3 APPLICATIONS TO SIMPLE MECHANISMS

Slider-Crank Inversions (Figure 1.14)

- Figure 1.14a with the driving crank

This mechanism falls into the second case category. The solutions are given by Equations 2.117 and 2.118. In this case $i = 1$, $j = 3$, $b_x = -r_2 \cos \theta_2$, $b_y = -r_2 \sin \theta_2$, $\dot{b}_x = r_2 \dot{\theta}_2 \sin \theta_2$, $\dot{b}_y = -r_2 \dot{\theta}_2 \cos \theta_2$, $r_2 = \text{const.}$, $r_3 = \text{const.}$, and $\theta_1 = \pi$. Assume also that the crank rotates with constant angular velocity, i.e., $\omega_2 = \text{const.}$ Then, $\ddot{b}_x = r_2 \omega_2^2 \cos \theta_2$, and $\ddot{b}_y = r_2 \omega_2^2 \sin \theta_2$. Taking all this into account, the general formulas, Equations 2.117 and 2.118, are reduced to

$$\alpha_3 = \frac{r_2 \omega_2^2 \sin \theta_2 + r_3 \omega_3^2 \sin \theta_3}{r_3 \cos \theta_3} \quad (2.135)$$

and

$$\ddot{i}_1 = \frac{1}{\cos \theta_3} [r_1 \omega_3 \sin \theta_3 - r_2 \omega_2 (\omega_2 - \omega_3) \cos(\theta_2 - \theta_3)] \quad (2.136)$$

The above equations can also be obtained by differentiating the expressions for the corresponding velocities (Equations 2.93 and 2.94).

A plot of the change of the angular acceleration of the connecting rod with the crank angle is shown in Figure 2.41. A change of the slider acceleration with the crank angle is shown in Figure 2.42.